# REZOLVĂRI PROBLEME PS

① a) $x[n] = x[-n]$

$$x[-n] \overset{Z}{\longleftrightarrow} \sum_{n=-\infty}^{\infty} x[-n]\, z^{-n} \overset{m=-n}{=\!=\!=} \sum_{m=-\infty}^{+\infty} x[m]\, z^{m} = \sum_{m=-\infty}^{\infty} x[m] (z^{-1})^{-m}$$

$$= X(z^{-1}) = X\left(\frac{1}{z}\right)$$

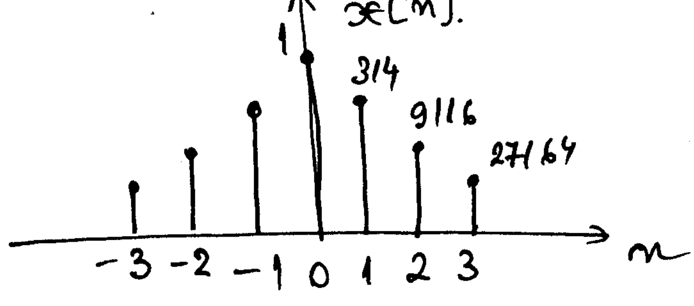
$$x[n] \longleftrightarrow X(z) \qquad \Rightarrow \qquad X(z) = X\left(\frac{1}{z}\right).$$

b) $$X(z) = \frac{(z-z_0)P(z)}{(z-z_p)Q(z)} \Rightarrow X\left(\frac{1}{z}\right) = \frac{\left(\frac{1}{z}-z_0\right)P\left(\frac{1}{z}\right)}{\left(\frac{1}{z}-z_p\right)Q\left(\frac{1}{z}\right)} = \frac{(1-z z_0)P\left(\frac{1}{z}\right)}{(1-z z_p)Q\left(\frac{1}{z}\right)}$$

$$= \frac{z_0}{z_p} \cdot \frac{\left(z-\frac{1}{z_0}\right)}{\left(z-\frac{1}{z_p}\right)} \frac{P\left(\frac{1}{z}\right)}{Q\left(\frac{1}{z}\right)}, \quad \frac{1}{z_0} - \text{zero}, \quad \frac{1}{z_p} - \text{pol}$$

⇒ zerourile lui $X(z)$ se pot grupa în perechi de forma $z_0, \frac{1}{z_0}$ iar polii în perechi $z_p, \frac{1}{z_p}$.

c) $x[n] = a^{|n|}$, $x[-n] = a^{|-n|} = a^{|n|} = x[n] \Rightarrow x[n]$ funcție pară




d) $$x[n] = \begin{cases} a^n, & n \geq 0 \\ a^{-n}, & n < 0 \end{cases} = a^n \sigma[n] + a^{-n}\sigma[-n-1] =$$

$$= a^n \sigma[n] + \left(\frac{1}{a}\right)^n \sigma[-n-1]$$

$$a^n \sigma[n] \overset{|z|>|a|}{\longleftrightarrow} \frac{1}{1-a z^{-1}} \quad \text{și} \quad -a^n \sigma[-n-1] \overset{|z|<|a|}{\longleftrightarrow} \frac{1}{1-a z^{-1}}$$

$$\Rightarrow -\left(\frac{1}{a}\right)^n \sigma[-n-1] \overset{|z| < \frac{1}{|a|}}{\Longleftrightarrow} \frac{1}{1-\frac{1}{a}z^{-1}} \Leftrightarrow \left(\frac{1}{a}\right)^n \sigma[-n-1] \overset{|z| < \frac{1}{|a|}}{\Longleftrightarrow} \frac{-1}{1-\frac{1}{a}z^{-1}}$$

$a = \frac{3}{4}$ $\Rightarrow$ regiunea de convergență este $\frac{3}{4} < |z| < \frac{4}{3}$

$$\left(\frac{3}{4}\right)^{|n|} \Leftrightarrow \frac{1}{1-\frac{3}{4}z^{-1}} - \frac{1}{1-\frac{4}{3}z^{-1}} = \frac{-\frac{7}{12}z^{-1}}{\left(1-\frac{3}{4}z^{-1}\right)\left(1-\frac{4}{3}z^{-1}\right)} = U(z)$$

$z_{p1} = \frac{3}{4}$, $z_{p2} = \frac{4}{3}$, $z_{01} = 0$

$\lim_{z \to \infty} U(z) = 0 \Rightarrow z_{02} = \infty$

② a) $s^2 Y(s) - 2sY(s) - 15Y(s) = X(s)$

$Y(s)(s^2 - 2s - 15) = X(s)$

$\frac{Y(s)}{X(s)} = \frac{1}{s^2 - 2s - 15} \Rightarrow H(s) = \frac{1}{s^2 - 2s - 15} = \frac{1}{(s+3)(s-5)}$

$\frac{1}{(s+3)(s-5)} = \frac{A}{s+3} + \frac{B}{s-5}$

$1 = As - 5A + Bs + 3B$

$A + B = 0 \rightarrow A = -B$

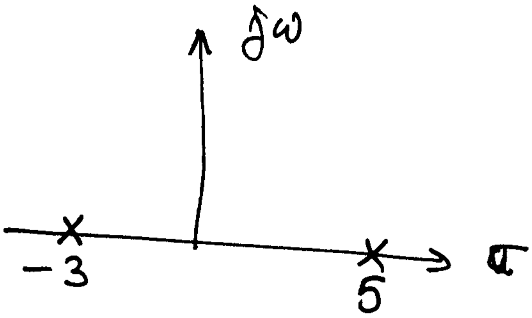
$3B - 5A = 1 \rightarrow 3B + 5B = 1 \Rightarrow 8B = 1 \rightarrow B = 1/8$

$A = -1/8$

$\Rightarrow H(s) = -\frac{1}{8} \cdot \frac{1}{s+3} + \frac{1}{8} \cdot \frac{1}{s-5}$

$s_{p1} = -3$

$s_{p2} = 5$




b) sistem stabil ⟺ DC: $-3 < Re\{s\} < 5$

$\Rightarrow h(t) = -\frac{1}{8} e^{-3t} \sigma(t) - \frac{1}{8} e^{5t} \sigma(-t)$

sistem cauzal ⟺ DC: $Re\{s\} > 5$

$h(t) = -\frac{1}{8} e^{-3t} \sigma(t) + \frac{1}{8} e^{5t} \sigma(t)$

sistem instabil, necauzal ⟺ DC: $Re\{s\} < -3$

$h(t) = \frac{1}{8} e^{-3t} \sigma(-t) - \frac{1}{8} e^{5t} \sigma(-t)$

3. $\frac{d^3y(t)}{dt^3} + (1+\alpha)\frac{d^2y(t)}{dt^2} + \alpha(1+\alpha)\frac{dy(t)}{dt} + \alpha^2 y(t) = x(t)$

a) $Y(s)\left[s^3+(1+\alpha)s^2+\alpha(\alpha+1)s+\alpha^2\right] = X(s)$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^3+(1+\alpha)s^2+\alpha(\alpha+1)s+\alpha^2} = \frac{1}{(s+1)(s^2+\alpha s+\alpha^2)}$$

b) $G(s) = sH(s) + H(s)$

$$G(s) = \frac{s+1}{s^3+(1+\alpha)s^2+\alpha(\alpha+1)s+\alpha^2} = \frac{1}{s^2+\alpha s+\alpha^2}$$

$\Rightarrow$ $G(s)$ are 2 poli

$H(s) \rightarrow 3$ poli : $-1$, $\left(-\frac{1}{2} \pm j\frac{\sqrt{3}}{2}\right)$

c) $\alpha = 2 \Rightarrow H(s) = \frac{1}{(s+1)(s^2+2s+4)}$

$$\frac{1}{(s+1)(s^2+2s+4)} = \frac{A}{s+1} + \frac{Bs+C}{s^2+2s+4}$$

$A(s^2+2s+4) + (Bs+C)(s+1) = 1$

$$\begin{cases} A+B=0 \\ 2A+B+C=0 \\ 4A+C=1 \end{cases} \Rightarrow \begin{cases} B=-A \\ 2A-A+C=0 \\ 4A+C=1 \end{cases} \Rightarrow \begin{cases} B=-A \\ A+C=0 \\ 4A+C=1 \end{cases}$$

$3A = 1 \Rightarrow A = \frac{1}{3}$

$B = -\frac{1}{3}$

$C = -\frac{1}{3}$

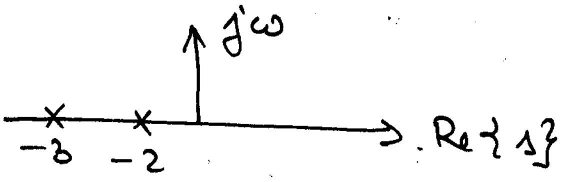
$$\Rightarrow H(s) = \frac{1/3}{s+1} + \frac{-\frac{1}{3}s-\frac{1}{3}}{s^2+2s+4} = \frac{1}{3}\cdot\frac{1}{s+1} - \frac{1}{3}\frac{s+1}{s^2+2s+4}$$

$\downarrow$

$$h(t) = \frac{1}{3}e^{t}\sigma(t) - \frac{1}{3}e^{-t}\cos\sqrt{2}t\ \sigma(t)$$

4. a) $x(t) = e^{-2t}\sigma(t) + e^{-3t}\sigma(t)$

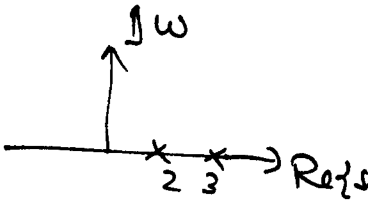
$$X(s) = \frac{1}{s+2} + \frac{1}{s+3}$$




DC: $Re\{s\} > -2$.

b) $x(t) = e^{2t}\sigma(-t) + e^{3t}\sigma(-t)$

$$X(s) = \frac{1}{s-2} + \frac{1}{s-3}, \quad Re\{s\} < 2$$




c) $x(t) = t\,e^{-2|t|} = t\,e^{2t}\sigma(-t) + t\,e^{-2t}\sigma(t)$

~~$X(s) = \frac{1}{(s-2)^2} + \frac{1}{s+2^2} = \frac{2s}{s^2-4}$~~

~~$-2 < Re\{s\} < 2$~~

$$e^{2t}\sigma(-t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s-2}$$

$$e^{-2t}\sigma(t) \overset{\mathcal{L}}{\longleftrightarrow} \frac{1}{s+2}$$

$$e^{-2|t|} \leftrightarrow \frac{1}{s+2} + \frac{1}{s-2} = \frac{s-2+s+2}{s^2-4} = \frac{2s}{s^2-4}$$

$$t e^{-2|t|} \leftrightarrow -\frac{d}{ds}\left[\frac{2s}{s^2-4}\right] = \frac{2s^2+8}{s^2-4}, \quad -2 < Re\{s\} < 2$$

d) $x(t) = \begin{cases} 1, & 0 \le t \le 1 \\ 0, & \text{rest} \end{cases}$

$$x(t) = \sigma(t) - \sigma(t-1)$$

$$\sigma(t) \leftrightarrow \frac{1}{s}$$

$$\sigma(t-1) \leftrightarrow \frac{e^{-s}}{s}$$

$$Re\{s\} > 0 \Rightarrow X(s) = \frac{1-e^{-s}}{s}$$

⑤ a) $[X(s) - kY(s)\cdot H_r(s)]\cdot H_d(s) = Y(s)$

$H_d(s)X(s) - kH_d(s)H_r(s)Y(s) = Y(s)$

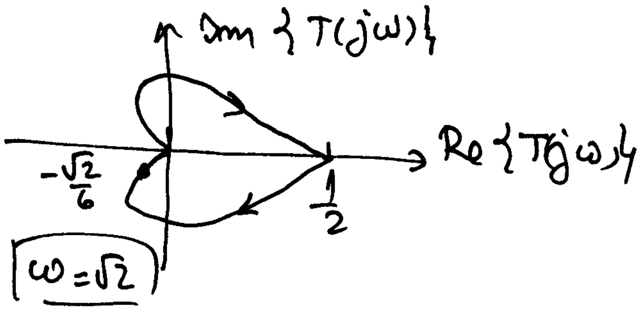
$$H(s) = \frac{Y(s)}{X(s)} = \frac{H_d(s)}{1+kH_d(s)H_r(s)} = \frac{\frac{1}{s+1}}{1+k\frac{1}{(s+1)(s+2)}}$$

b) $T(s) = H_d(s)\cdot H_r(s) = \frac{1}{(s+1)(s+2)}$

$$T(j\omega) = \frac{1}{(1+j\omega)(2+j\omega)} = \frac{(1-j\omega)(2-j\omega)}{(1+\omega^2)(4+\omega^2)} = \frac{2-3j\omega-\omega^2}{(1+\omega^2)(4+\omega^2)}$$

$$T(j\omega) = \underbrace{\frac{2-\omega^2}{(1+\omega^2)(4+\omega^2)}}_{Re\{T(j\omega)\}=R(\omega)} - j\underbrace{\frac{3\omega}{(1+\omega^2)(4+\omega^2)}}_{Im\{T(j\omega)\}=I(\omega)}$$

| $\omega$ | 0 | $\sqrt{2}$ | 2 | $\infty$ |
|---|---|---|---|---|
| $R(\omega)$ | 1/2 | 0 | −0,05 | 0 |
| $I(\omega)$ | 0 | $-\frac{\sqrt{2}}{6}$ | −0,15 | 0 |




$R(2) = -\frac{2}{5\cdot 8} = -\frac{1}{20} = -0{,}05$

$I(2) = -\frac{6}{5\cdot 8} = -\frac{3}{20} = -0{,}15$

$-\frac{1}{K} \geq \frac{1}{2}$ sau $-\frac{1}{K} < 0$

$\frac{1}{2} + \frac{1}{K} \leq 0 \Rightarrow \frac{K+2}{2K} \leq 0$

$K \in (-2, 0)$

$-\frac{1}{K} < 0 \Rightarrow K > 0.$ $\Rightarrow$ sistem stabil pentru $K \in (-2,0) \cup (0,+\infty)$

| | $-\infty$ | | −2 | | 0 | | $+\infty$ |
|---|---|---|---|---|---|---|---|
| $K+2$ | − − − | | 0 | + + + + | | + + | |
| $2K$ | − − − − | | | | 0 | + + + + | |
| $(K+2)/2K$ | + + | | 0 | − − | \| | + + + + | |

c) $$H(s) = \frac{H_d(s)}{1+kH_d(s)H_1(s)}$$

$$H(0) = \frac{H_d(0)}{1+kH_d(0)H_1(0)} = \frac{1}{1+k\cdot\frac{1}{2}} = \frac{1}{1+\frac{K}{2}} = \frac{2}{K+2}$$

$$H_d(0) \cdot H_h(0) = \frac{1}{(0+1)(0+2)} \Rightarrow H_d(0) H_h(0) = T(0)$$

$$\frac{H(0)}{H_d(0) H_1(0)} = \frac{\frac{2}{k+2}}{\frac{1}{2}} = \frac{4}{k+2} = 1 \Rightarrow \boxed{K=2}$$

6. a) $a_0=1$, $a_1=-2$, $a_2=1$

$b_0=1$

$$\Rightarrow H(z) = \frac{1}{1-2z^{-1}+z^{-2}} = \frac{1}{(1-z^{-1})^2} \Rightarrow z_{p1}^{-1}=z_{p2}^{-1}=1$$

Regiunile de convergență: $|z|<1$ sau $|z|>1$

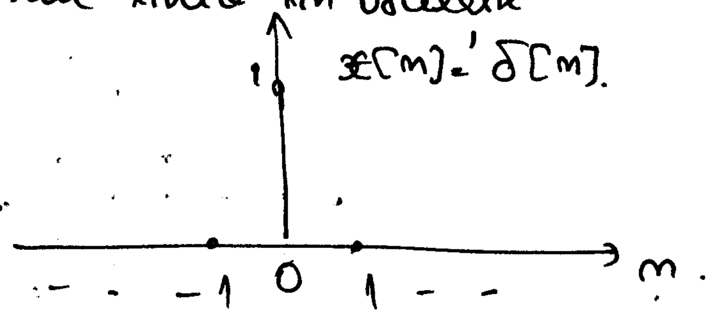
b) $H_1(z)=\frac{1}{1-z^{-1}} \overset{|z|>1}{\longleftrightarrow} \sigma[n]$

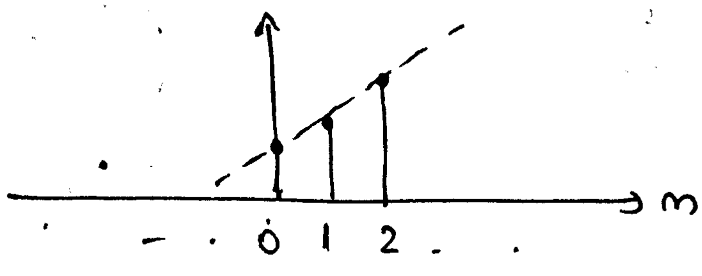
$$\frac{d}{dz}\{H_1(z)\} = \frac{d}{dz}\left\{\frac{z}{z-1}\right\} = \frac{z-1-z}{(z-1)^2} = \frac{-1}{(z-1)^2} = \frac{-z^{-2}}{(1-z^{-1})^2}$$

$$\Rightarrow \frac{1}{(1-z^{-1})^2} = -z^2\frac{d}{dz}\{H_1(z)\} \Leftrightarrow (n+1)\,\sigma[n+1]$$

Sistemul este stabil în sens strict. Dacă $x[n]=0 \Rightarrow y[n]=0$

$x[n]=\delta[n] \Rightarrow y[n]=h[n]=(n+1)\,\sigma[n+1]$ adică sistemul intră în oscilație



(7) a) $x_1(t) = x(t) \cdot \sin 8t$

$$X_1(\omega) = \frac{1}{2\pi}\left[X(\omega) * \frac{\pi}{j}\left(\delta(\omega-8) - \delta(\omega+8)\right)\right]$$

$$= \frac{1}{2j}\left[X(\omega-8) - X(\omega+8)\right]$$

$$jX_1(\omega) = \frac{X(\omega-8) - X(\omega+8)}{2}$$

$$X_2(\omega) = X_1(\omega) \cdot H_1(\omega) \Rightarrow jX_2(\omega) = jX_1(\omega)H_1(\omega)$$

$$h_1(t) = \frac{\sin 7t}{\pi t} - \frac{\sin 3t}{\pi t} \Rightarrow H_1(\omega) = p_7(\omega) - p_3(\omega)$$

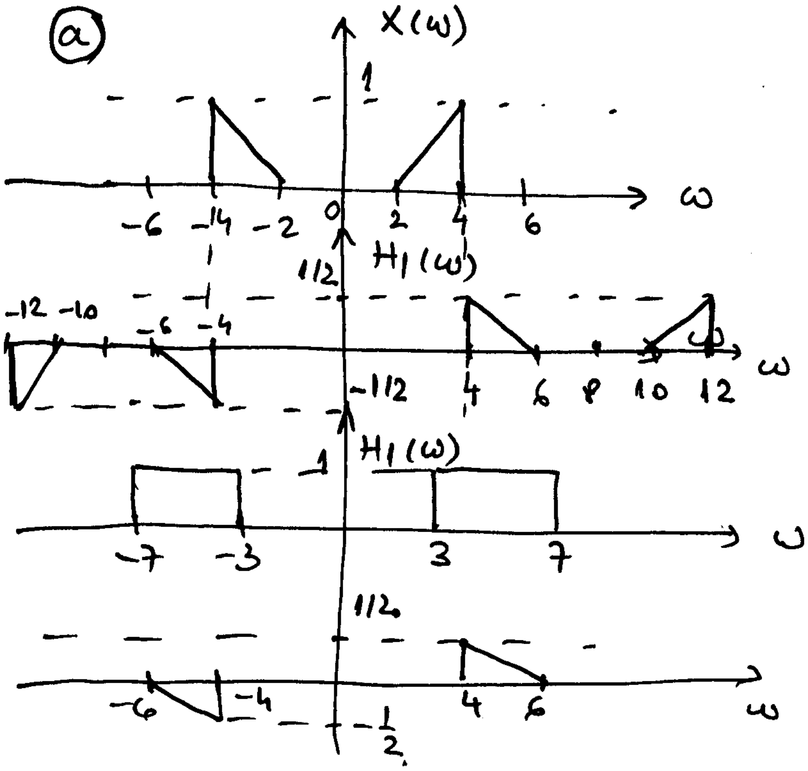
b) $x_3(t) = x_2(t) \cdot \sin 2t$

$$X_3(\omega) = \frac{1}{2j}\left[X_2(\omega-2) - X_2(\omega+2)\right] = \frac{jX_2(\omega+2) - jX_2(\omega-2)}{2}$$

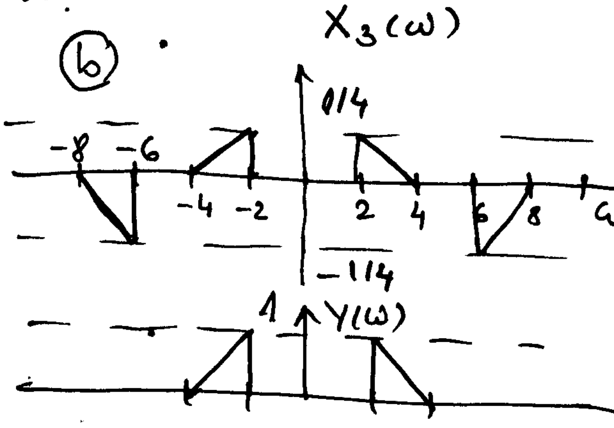
c) $$X_3(\omega) = \frac{jX_2(\omega+\omega_0) - jX_2(\omega-\omega_0)}{2}$$

$$\omega_0 = 8 \Rightarrow X_3(\omega) = \frac{jX_2(\omega+8) - jX_2(\omega-8)}{2}$$

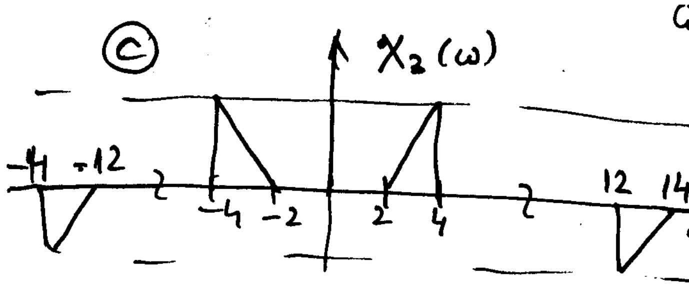
(a)




(b)




(c)



(8)

$$Y(s) = \frac{2}{s+2} - \frac{2}{s+4} = \frac{2(s+4)-2(s+2)}{(s+2)(s+4)} = \frac{4}{(s+2)(s+4)}$$

$$X(s) = \frac{2}{s+2} - \frac{1}{s+3} = \frac{2s+3-s-2}{(s+2)(s+3)} = \frac{s+1}{(s+2)(s+3)}$$

a) $$H(s) = \frac{Y(s)}{X(s)} = \frac{4}{(s+2)(s+4)} \cdot \frac{(s+2)(s+3)}{s+1} = \frac{4(s+3)}{(s+4)(s+1)}$$

$$H(\omega) = H(s)\Big|_{s=j\omega} = \frac{4(j\omega+3)}{(j\omega+4)(j\omega+1)}$$

b) $$H(s) = \frac{4(s+3)}{(s+4)(s+1)} = \frac{A}{s+4} + \frac{B}{s+1}$$

$$As + A + Bs + 4B = 4s + 12$$

$$\begin{cases} A + B = 4 \\ A + 4B = 12 \end{cases} \Rightarrow 3B = 8 \rightarrow B = 8/3 \Rightarrow A = 4 - \frac{8}{3} = \frac{4}{3}$$

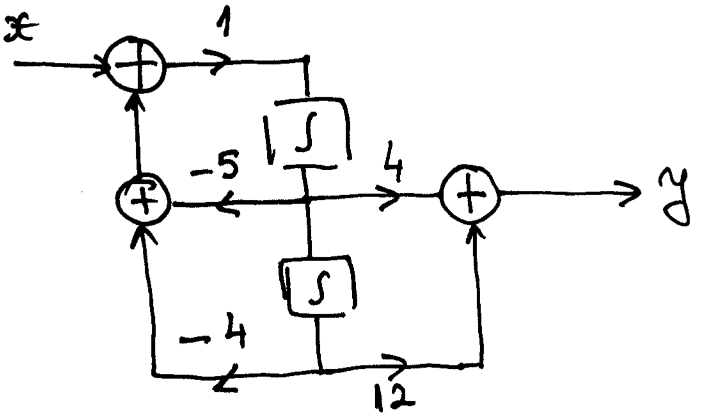
$$H(s) = \frac{4}{3} \cdot \frac{1}{s+4} + \frac{8}{3} \cdot \frac{1}{s+1}$$

$$h(t) = \frac{4}{3} e^{-4t} \sigma(t) + \frac{8}{3} e^{-t} \sigma(t).$$

c). $$H(s) = \frac{4s+12}{s^2+5s+4} \rightarrow \begin{matrix} a_0 = 4, & a_1 = 5; & a_2 = 1 \\ b_0 = 12, & b_1 = 4; & b_2 = 0 \end{matrix}$$

$\Rightarrow$ ec. diferențială: $y'' + 5y' + 4y = 4x' + 12x$

d)



(9) $y[n] - \frac{2}{3} y[n-1] + \frac{1}{9} y[n-2] = x[n]$

$$Y(z) - \frac{2}{3} z^{-1} Y(z) + \frac{1}{9} z^{-2} Y(z) = X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{2}{3} z^{-1} + \frac{1}{9} z^{-2}} = \frac{1}{\left(1 - \frac{1}{3} z^{-1}\right)^2}$$

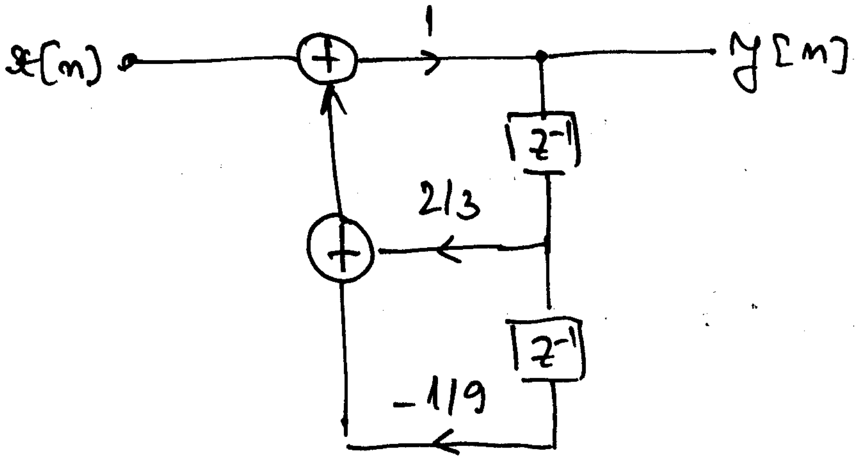
b) $H(z) = 3 \cdot \frac{\frac{1}{3} z^{-1}}{\left(1 - \frac{1}{3} z^{-1}\right)^2} \cdot z = \frac{3z \cdot \frac{1}{3} z^{-1}}{\left(1 - \frac{1}{3} z^{-1}\right)^2}$

$$h[n] = 3(n+1) \left(\frac{1}{3}\right)^{n+1} u[n+1]$$

c) $a_0 = 1$, $b_0 = 1$

$a_1 = -\frac{2}{3}$

$a_2 = \frac{1}{9}$



(10) a) ~~x(t) ...~~

$\frac{1}{a_0} = 1$ ; $-a_1 = \frac{5}{6}$ ; $-a_2 = -\frac{1}{6}$

$b_0 = 1$ ; $b_1 = \frac{-1}{2}$ ; $b_2 = 0$.

$\Rightarrow y[n] - \frac{5}{6} y[n-1] + \frac{1}{6} y[n-2] = x[n] - \frac{1}{2} x[n-1]. \quad |\mathcal{Z}$

$Y(z)\left[1 - \frac{5}{6} z^{-1} + \frac{1}{6} z^{-2}\right] = X(z)\left(1 - \frac{1}{2} z^{-1}\right).$

$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - \frac{1}{2} z^{-1}}{1 - \frac{5}{6} z^{-1} + \frac{1}{6} z^{-2}} = \frac{1 - \frac{1}{2} z^{-1}}{\left(1 - \frac{1}{2} z^{-1}\right)\left(1 - \frac{1}{3} z^{-1}\right)}$

$H(z) = \frac{1}{1 - \frac{1}{3} z^{-1}}, \quad |z| > \frac{1}{3}.$

$h[n] = \left(\frac{1}{3}\right)^n \sigma[n].$

b) $H(\Omega) = H(z)\big|_{z = e^{j\Omega}} = \frac{1}{1 - \frac{1}{3} e^{-j\Omega}}$

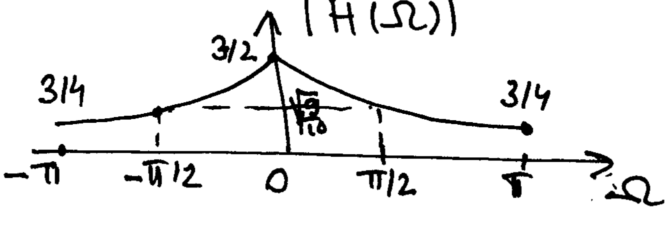
$H(\Omega) = \frac{1}{1 - \frac{1}{3}\cos\Omega + \frac{1}{3} j \sin\Omega}$

$|H(\Omega)| = \frac{1}{\sqrt{\left(1 - \frac{1}{3}\cos\Omega\right)^2 + \frac{1}{9}\sin^2\Omega}} = \frac{1}{\sqrt{1 - \frac{2}{3}\cos\Omega + \frac{1}{9}}}$

$= \frac{1}{\sqrt{\frac{10}{9} - \frac{2}{3}\cos\Omega}}.$ = ~~∠H~~

| $\Omega$ | $-\pi$ | $-\frac{\pi}{2}$ | $0$ | $\frac{\pi}{4}$ | $\frac{\pi}{2}$ | $\pi$ |
|---|---|---|---|---|---|---|
| $\lvert H(\Omega)\rvert$ | $3/4$ | $\sqrt{\frac{9}{10}}$ | $\frac{3}{2}$ | [illegible] | $\sqrt{\frac{9}{10}}$ | $\frac{3}{4}$ |

, $\left|H\left(\frac{\pi}{4}\right)\right| = \frac{1}{\sqrt{\frac{10}{9} - \frac{2}{3}\frac{\sqrt{2}}{2}}} = \frac{3}{\sqrt{10 - 3\sqrt{2}}}$




c) $\frac{10}{9}\left|H\left(\frac{\pi}{2}\right)\right| = \frac{10}{9} \cdot \frac{3}{\sqrt{10}} = \frac{10}{3\sqrt{10}} = \frac{10\sqrt{10}}{30}$

$= \frac{\sqrt{10}}{3}.$

(11) a) $s^2 Y(s) - (1+2\alpha) s Y(s) + (\alpha^2+\alpha-2) Y(s) = s X(s) + 4X(s)$

$Y(s) [s^2 - (1+2\alpha) s + (\alpha^2+\alpha-2)] = X(s)(s+4).$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s+4}{s^2-(1+2\alpha)s+\alpha^2+\alpha-2}.$$

b) Sistem cauzal ⇒ stabil dacă toți polii se găsesc în semiplanul stâng.

$s^2 - (1+2\alpha)s + \alpha^2+\alpha-2 = 0$

$\Delta = (1+2\alpha)^2 - 4(\alpha^2+\alpha-2) = 1+4\alpha+4\alpha^2-4\alpha^2-4\alpha+8 = 9$

$$s_{12} = \frac{1+2\alpha\pm 3}{2} \quad \begin{cases} s_{p1} = \alpha+2 \\ s_{p2} = \alpha-1 \end{cases}$$

$\alpha+2<0 \Rightarrow \alpha<-2$

$\alpha-1<0 \Rightarrow \alpha<1$

$\Rightarrow \boxed{\alpha<-2}$

c) $\alpha=-3 \Rightarrow \begin{matrix} s_{p1}=-1 \\ s_{p2}=-4 \end{matrix} \Rightarrow H(s) = \frac{s+4}{(s+1)(s+4)} = \frac{1}{s+1}.$

$Re\{s\} > -1.$

$H(j\omega) = \frac{1}{1+j\omega}$

$x(t) = \cos t$

$y(t) = |H(1)| \cos(t + \arg\{H(1)\})$

$|H(\omega)| = \frac{1}{\sqrt{1+\omega^2}} \Rightarrow |H(1)| = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2} \simeq 0,707.$

$\arg\{H(\omega)\} = -\text{arctg}\,\omega \rightarrow \arg\{H(1)\} = -\text{arctg}\,1 = -\frac{\pi}{4}$

$\Rightarrow y(t) = 0,707 \cos(t-\frac{\pi}{4}) = \frac{1}{\sqrt{2}}\cos(t-\frac{\pi}{4}).$

(12) a) $Y(z) - z^{-1}Y(z) + \frac{1}{4}Y(z) = 2X(z) - z^{-1}X(z)$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{2 - z^{-1}}{1 - z^{-1} + \frac{1}{4}z^{2}} = \frac{2(1 - \frac{1}{2}z^{-1})}{(1 - \frac{1}{2}z^{-1})^{2}} = \frac{2}{1 - \frac{1}{2}z^{-1}}$$

$$h[n] = 2 \cdot \left(\frac{1}{2}\right)^{n} \sigma[n].$$

b) $x[n] = \left(-\frac{1}{2}\right)^{n-1} \sigma[n-1].$

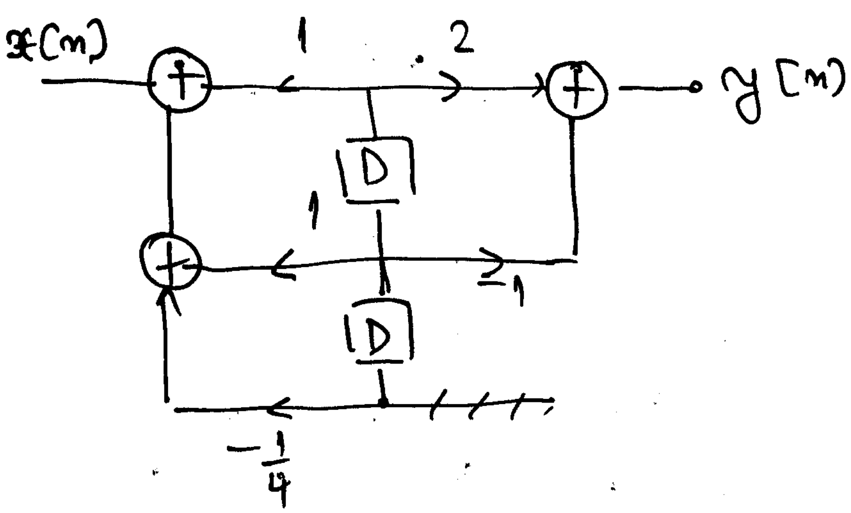
$$X(z) = \frac{z^{-1}}{1 + \frac{1}{2}z^{-1}} \Rightarrow Y(z) = X(z)H(z) = \frac{2z^{-1}}{(1+\frac{1}{2}z^{-1})(1-\frac{1}{2}z^{-1})}$$

$$\frac{2z^{-1}}{(1+\frac{1}{2}z^{-1})(1-\frac{1}{2}z^{-1})} = \frac{A}{1+\frac{1}{2}z^{-1}} + \frac{B}{1-\frac{1}{2}z^{-1}}$$

$$\begin{cases} A + B = 0 \\ (B-A)\frac{1}{2}z^{-1} = 2z^{-1} \end{cases} \Rightarrow \begin{matrix} A = -2 \\ B = 2. \end{matrix}$$

$$y[n] = -2\left(-\frac{1}{2}\right)^{n}\sigma[n] + 2\left(\frac{1}{2}\right)^{n}\sigma[n]$$

c) $a_0 = 1$; $a_1 = -1$; $a_2 = \frac{1}{4}$
$b_0 = 2$; $b_1 = -1$; $b_2 = 0$



(13) $x_1(t) = x(t)\cos 5\omega_0 t$.

$X_1(\omega) = \frac{1}{2\pi} X(\omega) * \pi[\delta(\omega-5\omega_0) + \delta(\omega+5\omega_0)]$.

$X_1(\omega) = \frac{1}{2} X(\omega-5\omega_0) + \frac{1}{2} X(\omega+5\omega_0)$

$X_2(\omega) = X_1(\omega)\cdot H_1(\omega)$.

$x_3(t) = x_2(t)\cdot \sin 3\omega_0 t$.

$X_3(\omega) = \frac{1}{2\pi} X_2(\omega) * \frac{\pi}{j}[\delta(\omega-3\omega_0) - \delta(\omega+3\omega_0)]$

$X_3(\omega) = \frac{1}{2j} X(\omega-3\omega_0) - \frac{1}{2j} X(\omega+3\omega_0)$

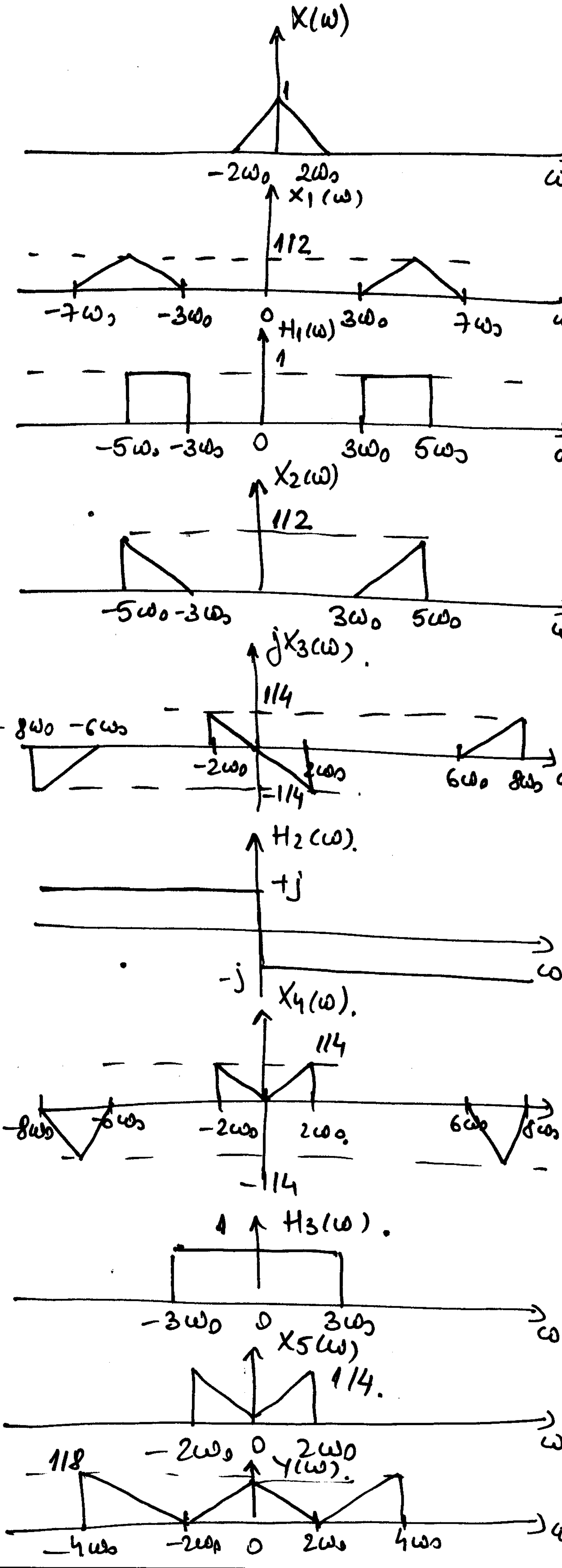
$X_4(\omega) = X_3(\omega)\cdot H_2(\omega)$.

$X_5(\omega) = X_4(\omega) H_3(\omega)$.

$y(t) = x_5(t)\cos 2\omega_0 t$

$Y(\omega) = \frac{1}{2\pi} X_5(\omega) * \pi[\delta(\omega-2\omega_0) + \delta(\omega+2\omega_0)]$

$Y(\omega) = \frac{1}{2} X_5(\omega-2\omega_0) + \frac{1}{2} X_5(\omega+2\omega_0)$.



(14) a) $X(z) = \dfrac{3z^2}{(z-\frac{1}{3})(z-\frac{1}{4})} = \dfrac{3\cancel{z^2}}{\cancel{z^2}(1-\frac{1}{3}z^{-1})(1-\frac{1}{4}z^{-1})}$

$$\frac{3}{(1-\frac{1}{3}z^{-1})(1-\frac{1}{4}z^{-1})} = \frac{A}{(1-\frac{1}{3}z^{-1})} + \frac{B}{(1-\frac{1}{4}z^{-1})}$$

$$A - \frac{1}{4}Az^{-1} + B - \frac{B}{3}z^{-1} = 3.$$

$$\begin{cases} A + B = 3 \\ \frac{1}{4}A - \frac{1}{3}B = 0 \end{cases} \Rightarrow \begin{cases} \frac{A}{3} + \frac{B}{3} = 1 \\ \frac{A}{4} - \frac{B}{3} = 0 \end{cases} \Rightarrow \frac{A}{3} + \frac{A}{4} = 1$$

$$\frac{4A + 3A}{12} = 1 \Rightarrow 7A = 12$$

$$A = \frac{12}{7} \Rightarrow B = 3 - \frac{12}{7} = \frac{21-12}{7} = \frac{9}{7}$$

$$\Rightarrow X(z) = \frac{12}{7} \cdot \frac{1}{1-\frac{1}{3}z^{-1}} + \frac{9}{7} \cdot \frac{1}{1-\frac{1}{4}z^{-1}}$$

$$x[n] = \frac{12}{7}\left(\frac{1}{3}\right)^n \sigma[n] + \frac{9}{7}\left(\frac{1}{4}\right)^n \sigma[n].$$

b) $X(z) = \dfrac{2/9}{1-z^{-1}} + \dfrac{7/9}{1+2z^{-1}}$

$$x[n] = \frac{2}{9}\sigma[n] + \frac{7}{2}(-2)^n \sigma[n].$$

c). $X(z) = 4 - 4z^{-2} + 4z^{-3} - 4z^{-5} \Rightarrow 4\,\delta[n] - 4\,\delta[n-2] +$
$+ 4\,\delta[n-3] - 4\,\delta[n-5].$

d) $X(z) = \dfrac{1}{(1-\frac{1}{4}z^{-1})^2} \Rightarrow X_1(z) = \dfrac{1}{1-\frac{1}{4}z^{-1}} \Rightarrow \dfrac{d}{dz}(X_1(z)) =$

–

$$X_1(z) = \frac{d}{dz}\left(\frac{1}{1-\frac{1}{4}z^{-1}}\right) = \frac{d}{dz}\left(\frac{4z}{4z-1}\right) = \frac{16z - 4 - 16z}{(4z-1)^2} = \frac{-4}{(4z-1)^2}.$$

$$-\frac{dX_1(z)}{dz} = \frac{1/4}{\left(z-\frac{1}{4}\right)^2}$$

$$-z^2\frac{dX_1(z)}{dz} = \frac{1/4\, z^2}{\left(z-\frac{1}{4}\right)^2} = \frac{\frac{1}{4}}{\left(1-\frac{1}{4}z^{-1}\right)^2}$$

$$X(z) = 4\left[z^{-2}\frac{dX_1(z)}{dz}\right] = -\frac{z^2}{1/4}\frac{dX_1(z)}{dz}.$$

$$x_1[n] = \left(\frac{1}{4}\right)^n u[n]$$

$$-z\frac{dX_1(z)}{dz} \Leftrightarrow n\,x_1[n] \Rightarrow \frac{z}{1/4}\left(-z\frac{dX_1(z)}{dz}\right) \Leftrightarrow 4(n+1)\,x_1[n+1]$$

$$\Rightarrow x[n] = 4(n+1)\left(\frac{1}{4}\right)^{n+1} u[n+1].$$

$$x[n] = \left(\frac{1}{4}\right)^n (n+1)\; u[n+1].$$

(15) a) $$H(z) = \frac{1 - z^{-1}}{1 - 2z^{-1} + 0{,}75 z^{-2}} = \frac{1 - \frac{1}{z}}{1 - \frac{2}{z} + \frac{3}{4z^2}} = \frac{z(z-1)}{z^2 + 2z + 3/4}$$

$$\Delta = 1 \Rightarrow \begin{cases} z_1 = -3/2 \\ z_2 = -1/2. \end{cases}$$

$$\Rightarrow H(z) = \frac{z(z-1)}{(z+3/2)(z+\frac{1}{2})} = \frac{(1 - z^{-1})}{(1+\frac{3}{2}z^{-1})(1+\frac{1}{2}z^{-1})}.$$

$$\frac{1 - z^{-1}}{(1+\frac{3}{2}z^{-1})(1+\frac{1}{2}z^{-1})} = \frac{A}{1+\frac{3}{2}z^{-1}} + \frac{B}{1+\frac{1}{2}z^{-1}}$$

$$\begin{cases} A + B = 1 \\ \frac{A}{2} + \frac{3B}{2} = -1 \end{cases} \rightarrow \begin{cases} A + B = 1 \\ A + 3B = -2 \end{cases} \rightarrow 2B = -3 \rightarrow B = -\frac{3}{2}$$

$$A = 1 - B = 1 + \frac{3}{2} = \frac{5}{2}.$$

$$\Rightarrow H(z) = \frac{5}{2} \cdot \frac{1}{1+\frac{3}{2}z^{-1}} - \frac{3}{2} \frac{1}{1+\frac{1}{2}z^{-1}}$$

$z_{p1} = -3/2$, $z_{01} = 0$

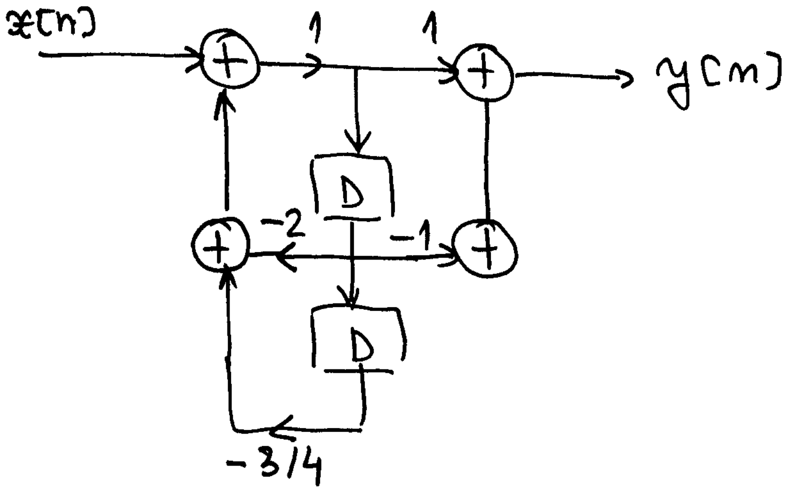
$z_{p2} = -1/2$, $z_{02} = 1$

b) $$h[n] = \frac{5}{2}\left(-\frac{3}{2}\right)^n \sigma[n] - \frac{3}{2}\left(-\frac{1}{2}\right)^n \sigma[n].$$

c) $a_0 = 1$ $b_0 = 1$

$a_1 = 2$ $b_1 = -1$

$a_2 = 3/4.$ $b_2 = 0$



(16) $$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

$$sY(s) + 2Y(s) = X(s)$$

$$Y(s)(s+2) = X(s) \Rightarrow \frac{Y(s)}{X(s)} = \frac{1}{s+2}$$

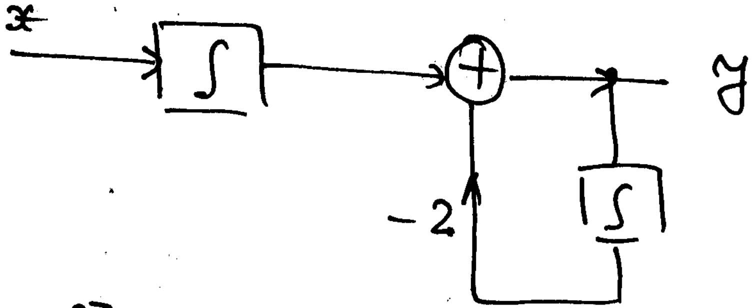
$$H_c(s) = \frac{1}{s+2} \Rightarrow h_c(t) = e^{-2t}\sigma(t).$$

$$H_c(j\omega) = H_c(s)\Big|_{s=j\omega} = \frac{1}{j\omega+2}$$

b) $$h_d[n] = e^{-2nT}\sigma[n]$$

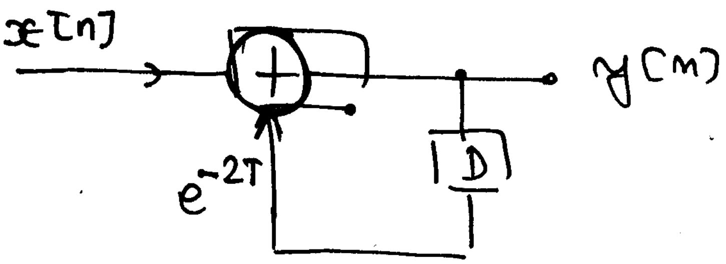
$$H_d(z) = \frac{1}{1-e^{-2t}z^{-1}}.$$

c) sistem analogic




$$b_0 = 1;\ a_0 = 1;\ a_1 = -e^{-2T}$$

$$y[n] - e^{-2T}y[n-1] = x[n]$$



(17)

a) $r_1(t) = x(t)\cos(10\omega_0 t) \rightarrow R_1(\omega) = \frac{1}{2}\left[X(\omega - 10\omega_0) + X(\omega + 10\omega_0)\right]$

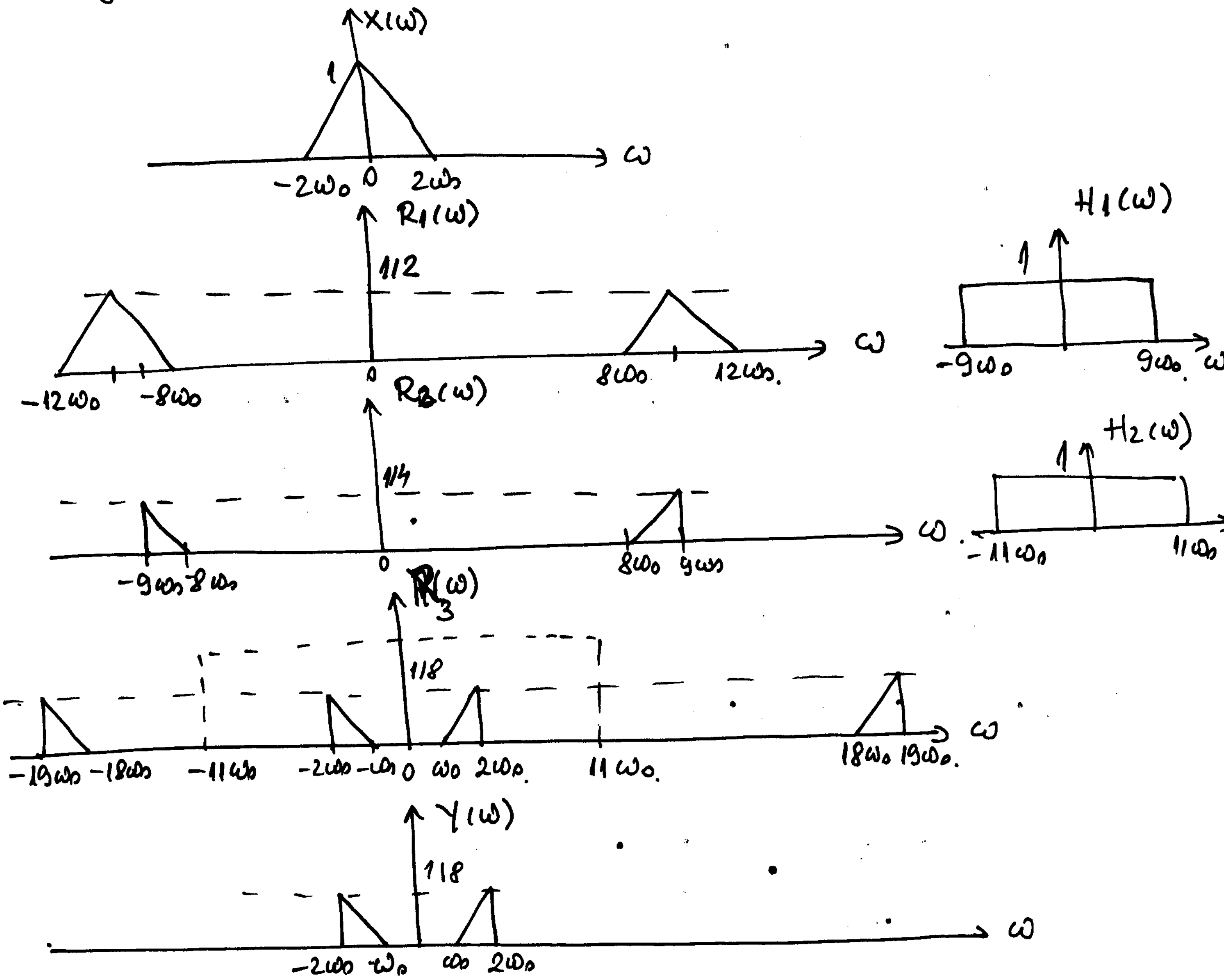
$r_2(t) = r_1(t) * h_1(t) \Rightarrow R_2(\omega) = R_1(\omega)\cdot H_1(\omega)$

$$h_1(t) = \frac{\sin 9\omega_0 t}{\pi t} \longleftrightarrow p_{9\omega_0}(\omega) = \begin{cases} 1, & |\omega| < 9\omega_0 \\ 0 & |\omega| > 9\omega_0 \end{cases}$$

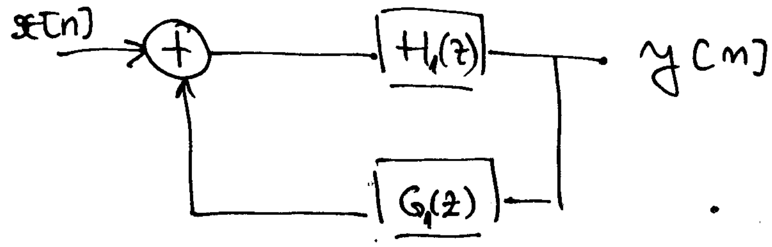
$$h_2(t) = \frac{\sin 11\omega_0 t}{\pi t} \Leftrightarrow p_{11\omega_0}(\omega) = \begin{cases} 1, & |\omega| < 11\omega_0 \\ 0, & |\omega| > 11\omega_0 \end{cases}$$

$r_3(t) = r_2(t)\cdot\cos(10\omega_0 t) \rightarrow R_3(\omega) = \frac{1}{2}\left[R_2(\omega - 10\omega_0) + R_2(\omega + 10\omega_0)\right]$

$Y(\omega) = R_3(\omega)\cdot H_2(\omega)$



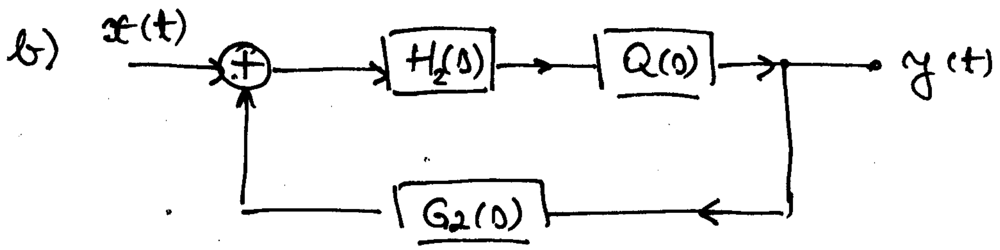
(18) a) Sistemul poate fi văzut ca interconectarea în paralel a sistemului cu funcția $H_0(s)$ cu sistemul din figură.




$$\rightarrow Q(z) = \frac{Y(z)}{X(s)} = \frac{H_1(z)}{1+G(z)H_1(z)}$$

$\Rightarrow$ funcția întregului sistem: $Q_1(z) = Q(z) + H_0(z) = H_0(z) + \dfrac{H_1(z)}{1+G(z)H_1(z)}$

b)

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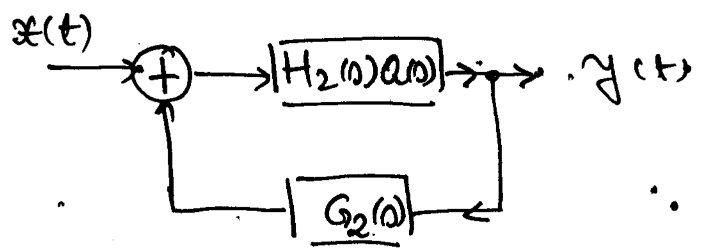



$$Q(s) = \frac{H_1(s)}{1+G_1(s)H_1(s)}$$

$$Q_1(s) = \frac{\dfrac{H_2(s)\,H_1(s)}{1+G_1(s)H_1(s)}}{1+\dfrac{G_2(s)H_2(s)H_1(s)}{1+G_1(s)H_1(s)}} = \frac{H_1(s)\,H_2(s)}{1+G_1(s)H_1(s)+G_2(s)H_1(s)H_2(s)}$$

(fracția $\dfrac{H_1(s)}{1+G_1(s)H_1(s)}$ este marcată $= Q(s)$)

(19.) $z(t) = x_1(t)\, e^{j\omega_c t} \Leftrightarrow Z(\omega) = X_1(\omega - \omega_c)$.

$\text{sup}\{Z(\omega)\} = \text{sup}\{X_1(\omega-\omega_c)\}$.

$\text{sup}\{Z(\omega)\} = [\omega_1, \omega_2]$

$\text{sup}\{X_1(\omega)\} = [-\omega_M, \omega_M]$.

$\text{sup}\{X_1(\omega-\omega_c)\} = [-\omega_M+\omega_c, \omega_M+\omega_c]$

$\Leftrightarrow -\omega_M + \omega_c = \omega_1$ și
$\omega_M + \omega_c = \omega_2$ (adevărat)

b) $x_1(t) = z(t)\, e^{-j\omega_c t}$

→ semnalul recuperat $x_1(t) = \sum_{k=-\infty}^{\infty} x_1(kT_e) \dfrac{\sin\left(\pi\left(\frac{t}{T_e}-k\right)\right)}{\pi\left(\frac{t}{T_e}-k\right)}$

dacă $\omega_c = \omega_M = \dfrac{\omega_e}{2}$

$x_1(t) = \sum_{-\infty}^{+\infty} x_1(kT_e)\, \text{sinc}\left(\pi\left(\frac{t}{T_e}-k\right)\right)$

$z(t) = x_1(t)\, e^{j\omega_c t} = \sum_{-\infty}^{+\infty} e^{j\omega_c t}\, x_1(kT_e)\, \text{sinc}\left(\pi\left(\frac{t}{T_e}-k\right)\right)$.

$\Rightarrow z(t) = \sum_{k=-\infty}^{+\infty} e^{j\omega_c(t-kT_e)}\, z(kT_e)\, \text{sinc}\left(\pi\left(\frac{t}{T_e}-k\right)\right), \quad \omega_c = \omega_M = \frac{\omega_e}{2}$

și $T_e = \dfrac{\pi}{\omega_M}$

(28.) a) $x_2(t) = x_1(t)\cos 3\pi t$

$$X_2(\omega) = \frac{1}{2\pi} X_1(\omega) * \pi[\delta(\omega - 3\pi) + \delta(\omega + 3\pi)]$$

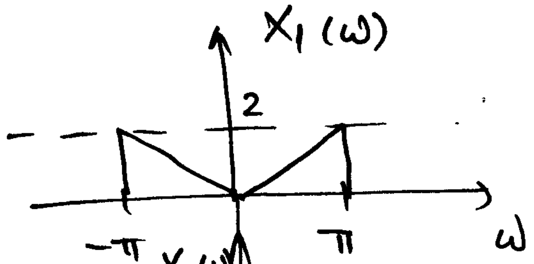
$$= \frac{1}{2} X_1(\omega - 3\pi) + \frac{1}{2} X_1(\omega + 3\pi)$$

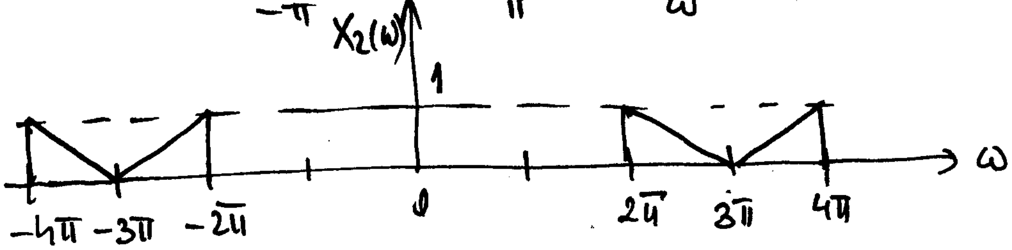
$x_5(t) = x_4(t)\cos 5\pi t$

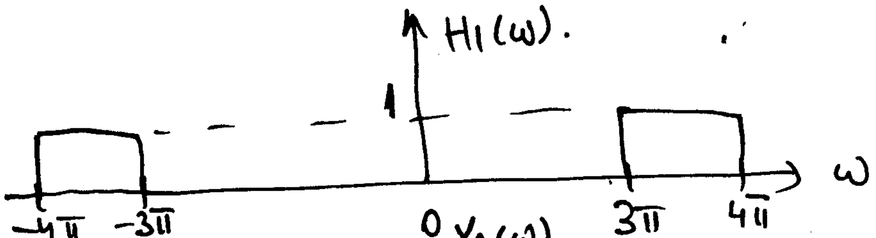
$$X_5(\omega) = \frac{1}{2\pi} X_4(\omega) * \pi[\delta(\omega - 5\pi) + \delta(\omega + 5\pi)]$$

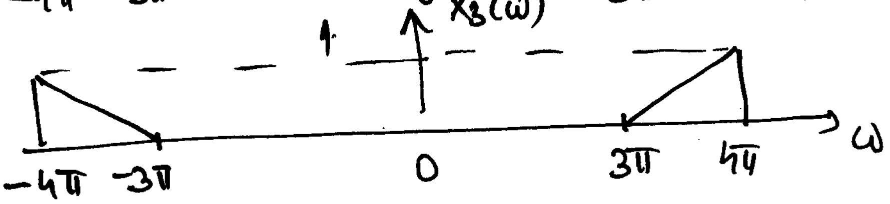
$$= \frac{1}{2} X_4(\omega - 5\pi) + \frac{1}{2} X_4(\omega + 5\pi)$$

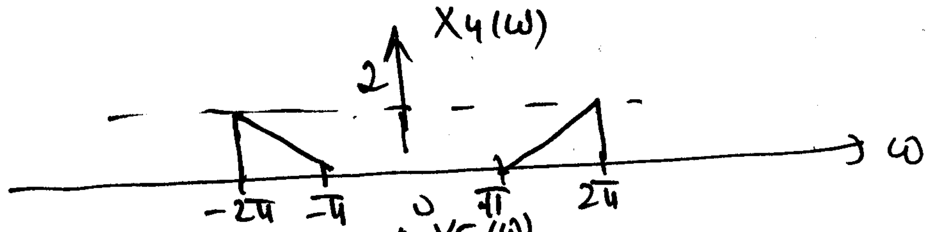
$x_3(t) = x_2(t) * h_1(t) \rightarrow X_3(\omega) = X_2(\omega) H_1(\omega)$

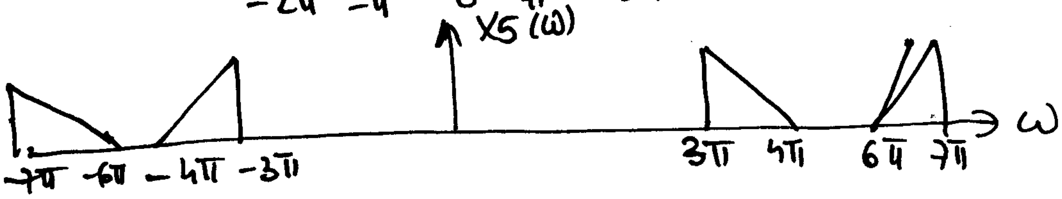


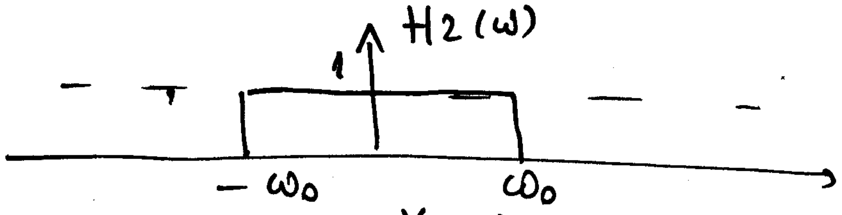



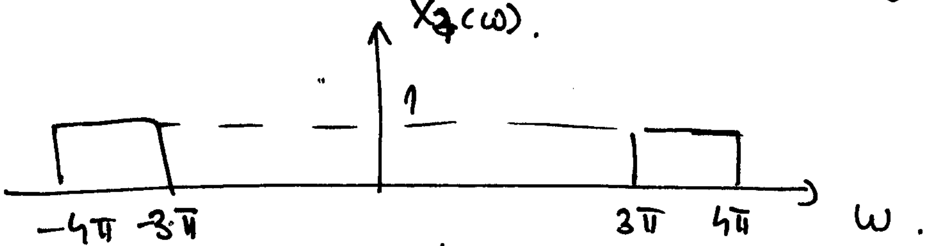



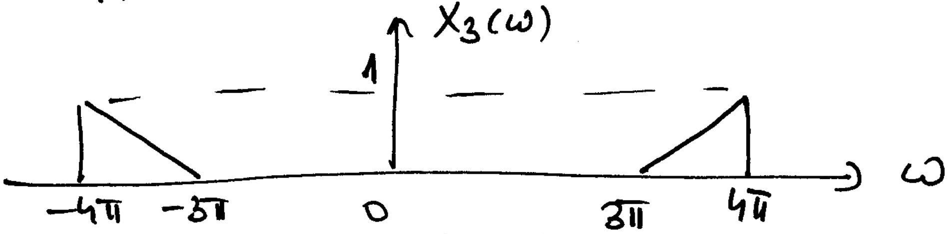
b) $x_6(t) = x_5(t) * h_2(t)$

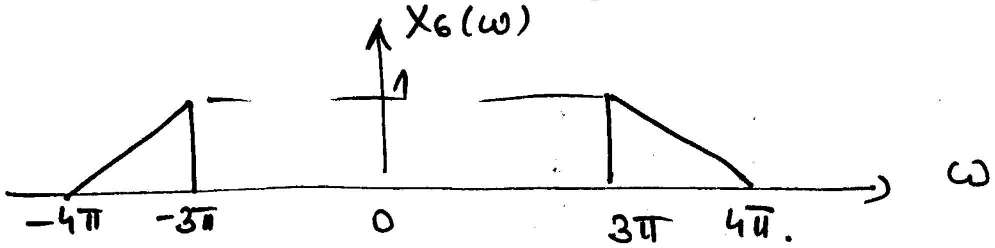
$X_6(\omega) = X_5(\omega) \cdot H_2(\omega)$

$$x_7(t) = \frac{\sin 4\pi t - \sin 3\pi t}{\pi t} \Rightarrow x_7(t) = P_{4\pi}(\omega) - P_{3\pi}(\omega).$$



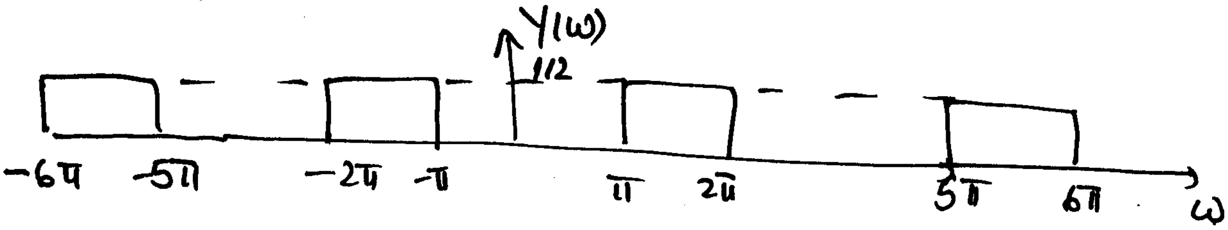




$\Rightarrow$ $\underline{\omega_{0\,min} = 4\pi}$

c) $y(t) = x_7(t) \cos 2\pi t$

$$Y(\omega) = \frac{1}{2\pi} X_7(\omega) * \pi\left[\delta(\omega - 2\pi) + \delta(\omega + 2\pi)\right] =$$

$$= \frac{1}{2\pi} X_7(\omega - 2\pi) + \frac{1}{2} X_7(\omega + 2\pi).$$




$\omega_M = 6\pi \Rightarrow f_M = \frac{\omega_M}{2\pi} = \frac{6\pi}{2\pi} = 3\,[Hz].$

$f_e \geq 2 f_M \Rightarrow \underline{f_{e\,min} = 6\,Hz}$

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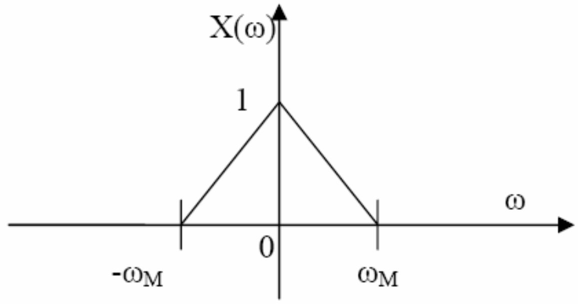



a) $p(t)=\sum_{n=-\infty}^{\infty} a_n e^{jn\frac{2\pi}{T}t}$

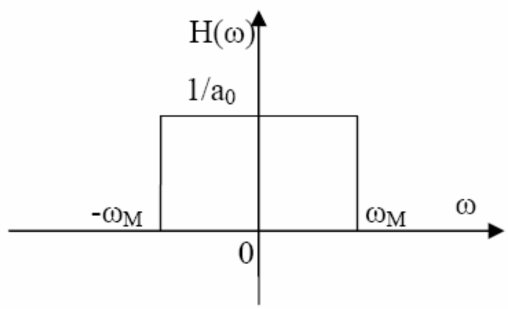
$$P(\omega)=\sum_{n=-\infty}^{\infty} 2\pi a_n \delta\left(\omega-n\frac{2\pi}{T}\right)$$

$$X_e(\omega)=\frac{1}{2\pi}X(\omega)*P(\omega)$$

$$X_e(\omega)=\sum_n a_n X\left(\omega-n\frac{2\pi}{T}\right),$$

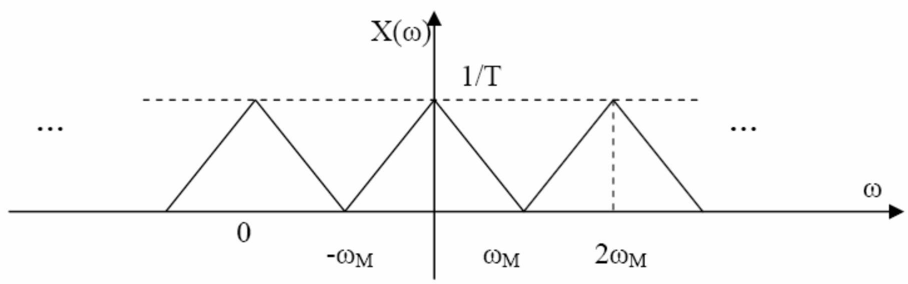
$$\omega_e=\frac{2\pi}{T}=2\omega_M$$

b) $a_0 \neq 0 \Rightarrow X_e(\omega)=a_0X(\omega)+\sum_{n\neq 0} a_n X\left(\omega-n\frac{2\pi}{T}\right)$




d) p(t)=$\delta_T$(t); $P(\omega)=\frac{2\pi}{T}\delta_{\frac{2\pi}{T}}(\omega)$

$$X_e(\omega)=\frac{1}{2\pi}X(\omega)*P(\omega)=\frac{1}{T}\sum X\left(\omega-k\frac{2\pi}{T}\right)$$




p(t)=$\delta_T$(t-Δ); $P(\omega)=e^{-j\omega\Delta}\frac{2\pi}{T}\delta_{\frac{2\pi}{T}}(\omega)$

$$X_e(\omega)=\frac{1}{2\pi}X(\omega)*P(\omega)=\frac{1}{2\pi}X(\omega)*e^{-j\omega\Delta}\frac{2\pi}{T}\sum_k \delta_{\frac{2\pi}{T}}\left(\omega-k\frac{2\pi}{T}\right)$$

$$X_e(\omega)=\frac{1}{T}X(\omega)*\sum_k \delta\left(\omega-k\frac{2\pi}{T}\right)e^{-jk\frac{2\pi}{T}\Delta}=\frac{1}{T}\sum_k X\left(\omega-k\frac{2\pi}{T}\right)e^{-jk\frac{2\pi}{T}\Delta}$$

$$\left|X_e(\omega)\right|=\frac{1}{T}\sum_k \left|X\left(\omega-k\frac{2\pi}{T}\right)\right|$$

**Cod: E 01**

**REZOLVARE**

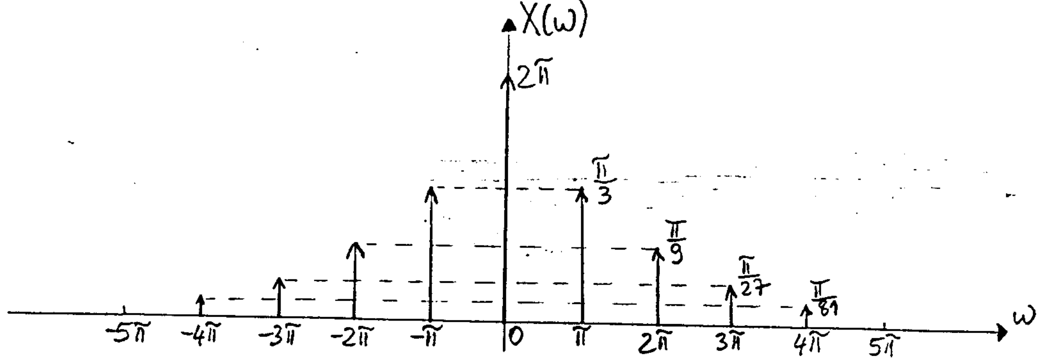
a) $x(t) = \sum_{k=0}^{4} \left(\frac{1}{3}\right)^k \cos(k\pi t)$

$$x(t) \xleftrightarrow{\mathcal{F}} X(\omega)$$

$$\cos \omega_0 t \xleftrightarrow{\mathcal{F}} \pi[\delta(\omega-\omega_0) + \delta(\omega+\omega_0)]$$

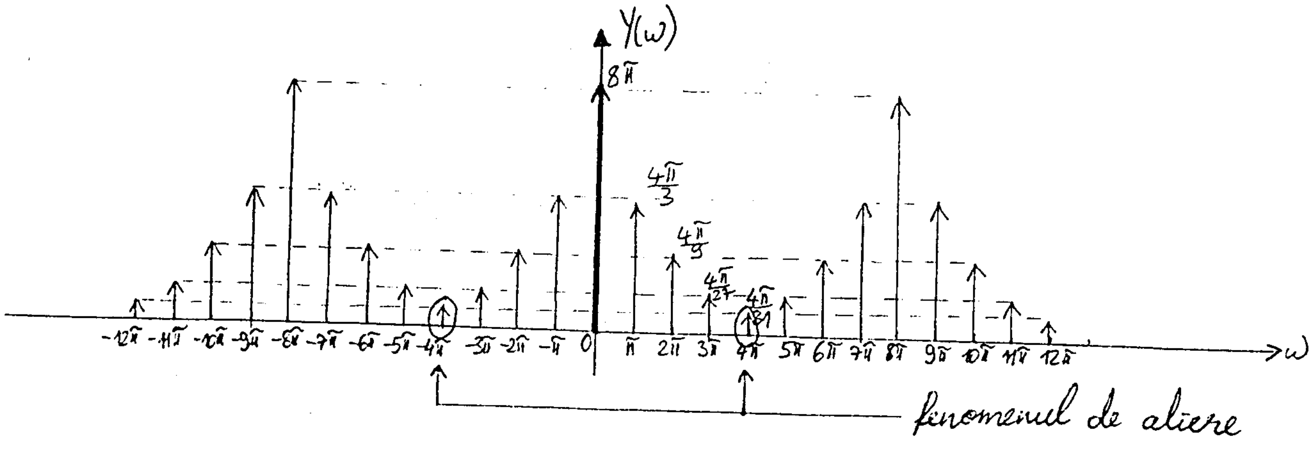
$$\mathcal{F}\{x(t)\}(\omega) = \sum_{k=0}^{4} \left(\frac{1}{3}\right)^k \pi[\delta(\omega-k\pi) + \delta(\omega+k\pi)]$$

$$X(\omega) = \pi \sum_{k=0}^{4} \left(\frac{1}{3}\right)^k [\delta(\omega-k\pi) + \delta(\omega+k\pi)]$$




b) $Y(\omega) = \frac{1}{T_e} \sum_{k=-\infty}^{\infty} X\left(\omega - k\frac{2\pi}{T_e}\right)$ $\qquad T_e = \frac{1}{f_e} = \frac{1}{4} = 0{,}25$

$$Y(\omega) = 4 \sum_{k=-\infty}^{\infty} X(\omega - 8k\pi)$$




c) Deoarece pulsaţia maximă din spectrul semnalului de eşantionare este $4\pi$, iar pulsaţia de eşantionare este $8\pi$, apare fenomenul de aliere.

(23.) $x(t) = \sin 200\pi t + 2\sin 400\pi t$

$$X(\omega) = \frac{\pi}{j}\left[\delta(\omega-200\pi) - \delta(\omega+200\pi)\right] + \frac{2\pi}{j}\left[\delta(\omega-400\pi) - \delta(\omega+400\pi)\right]$$

$$g(t) = x(t)\cdot \sin 400\pi t$$

$$G(\omega) = \frac{1}{2\pi} X(\omega) * \frac{\pi}{j}\left[\delta(\omega-400\pi) - \delta(\omega+400\pi)\right]$$

$$= \frac{1}{2j} X(\omega-400\pi) - \frac{1}{2j} X(\omega+400\pi)$$

$$= \frac{\pi}{j}\cdot\frac{1}{2j}\left[\delta(\omega-600\pi) - \delta(\omega-200\pi)\right] + \frac{2\pi}{j}\cdot\frac{1}{2j}\left[\delta(\omega-800\pi) - \delta(\omega)\right]$$

$$- \frac{1}{2j}\cdot\frac{\pi}{j}\left[\delta(\omega+200\pi) - \delta(\omega+600\pi)\right] - \frac{2\pi}{j}\cdot\frac{1}{2j}\left[\delta(\omega) - \delta(\omega+800\pi)\right]$$

$$G(\omega) = -\frac{\pi}{2}\left[\delta(\omega-600\pi) - \delta(\omega-200\pi)\right] - \pi\left[\delta(\omega-800\pi) - \delta(\omega)\right]$$

$$+ \frac{\pi}{2}\left[\delta(\omega+200\pi) - \delta(\omega+600\pi)\right] + \pi\left[\delta(\omega) - \delta(\omega+800\pi)\right]$$

$$g'(t) = g(t)\cdot \sin 400\pi t$$

$$G'(\omega) = \frac{1}{2\pi} G(\omega) * \frac{\pi}{j}\left[\delta(\omega-400\pi) - \delta(\omega+400\pi)\right].$$

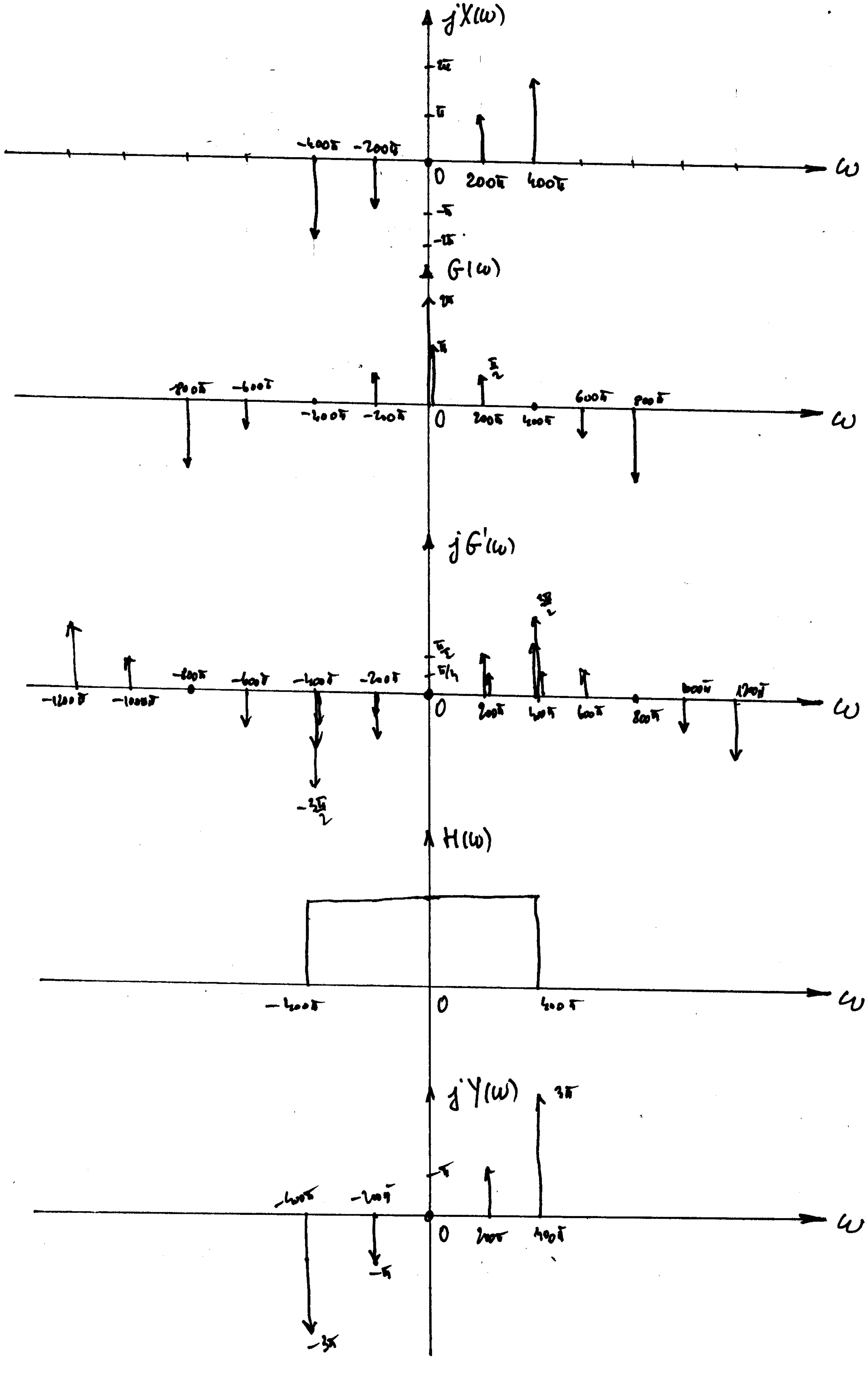
$$= \frac{1}{2j} G(\omega-400\pi) - \frac{1}{2j} G(\omega+400\pi).$$

$$G'(\omega) = \frac{1}{2j}\cdot\left(-\frac{\pi}{2}\right)\cdot\left[\delta(\omega-1000\pi) - \delta(\omega-600\pi)\right] - \pi\cdot\frac{1}{2j}\left[\delta(\omega-1200\pi) - \delta(\omega-400\pi)\right]$$

$$+ \frac{\pi}{2}\cdot\frac{1}{2j}\left[\delta(\omega-200\pi) - \delta(\omega+200\pi)\right] + \pi\cdot\frac{1}{2j}\left[\delta(\omega-400\pi) - \delta(\omega+400\pi)\right]$$

$$- \frac{1}{2j}\cdot\left(-\frac{\pi}{2}\right)\left[\delta(\omega-200\pi) - \delta(\omega+200\pi)\right] - \pi\cdot\left(-\frac{1}{2j}\right)\left[\delta(\omega-400\pi) - \delta(\omega+400\pi)\right]$$

$$+ \frac{\pi}{2}\left(-\frac{1}{2j}\right)\left[\delta(\omega+600\pi) - \delta(\omega+1000\pi)\right] + \pi\left(-\frac{1}{2j}\right)\left[\delta(\omega+400\pi) - \delta(\omega+1200\pi)\right]$$

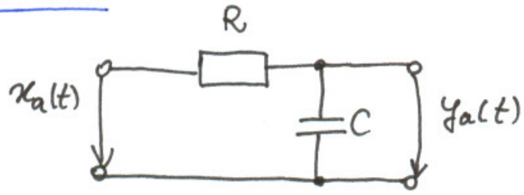


jX(ω)
-400π
-200π
0
200π
400π
ω
G(ω)
-800π
-600π
-400π
-200π
0
200π
400π
600π
800π
ω
jG'(ω)
-1200π
-1000π
-800π
-600π
-400π
-200π
0
200π
400π
600π
800π
1000π
1200π
ω
H(ω)
-400π
0
400π
ω
jY(ω)
3π
-400π
-200π
0
200π
400π
-π
-3π
ω

REZOLVARE

1p start

a) 2p




$$Y_a(\omega)=\frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}}\cdot X_a(\omega)\Rightarrow\frac{Y_a(\omega)}{X_a(\omega)}=\frac{\frac{1}{j\omega C}}{\frac{j\omega RC+1}{j\omega C}}=\frac{1}{1+j\omega RC}\quad 1p.$$

Deci: $H_a(\omega)=\frac{1}{1+j\omega RC}$ 0.5p ; $H_a(s)=H_a(\omega)\Big|_{s=j\omega}=\frac{1}{1+sRC}$ 0.5p

b) 2p

$$\omega_0=\frac{1}{RC}=\frac{1}{100\cdot10^3\cdot0{,}01\cdot10^{-6}}=1000\ rad/s\quad 0{,}66p$$

$$\tau=RC=100\cdot10^3\cdot0{,}01\cdot10^{-6}=1ms.\quad 0{,}66p$$

$$f_0=\frac{\omega_0}{2\pi}=\frac{1000}{6{,}28}\cong159\ Hz.\quad 0{,}67p$$

c) 2p

$$H_a(s)=\frac{1}{1+sRC}=\frac{1}{RC}\cdot\frac{1}{\frac{1}{RC}+s}\ \text{0,5p}\ \Longleftrightarrow h_a(t)=\frac{1}{RC}\cdot e^{-\frac{1}{RC}t}\sigma(t).\quad 1{,}5p$$

d) 3p Pentru tipul de echivalare cerut:

$$h_d[n]=T_e\cdot h_a(nT_e)\quad 1p.$$

$$h_d[n]=\frac{1}{RC}\cdot T_e\, e^{-\frac{1}{RC}nT_e}\sigma(nT_e)=\frac{1}{RC}\cdot T_e\cdot e^{-\frac{1}{RC}nT_e}\sigma[n].\quad 1p.$$

$R=100k\Omega$, $C=0{,}01\mu F$, $T_e=0{,}1ms$

$$h_d[n]=0{,}1\,e^{-0{,}1n}\sigma[n].$$

$$H_d(z)=\frac{0{,}1}{1-e^{0{,}1}\cdot z^{-1}}.\quad 1p.$$

(25.)

a) $H(s) = \dfrac{Y(s)}{X(s)} = \dfrac{H_d(s)}{1 + K \cdot H_d(s) \cdot H_r(s)} = \dfrac{H_d(s)}{1 + K \cdot \dfrac{s+1}{s^2-4}}$.

b) Notăm: $T(s) = H_d(s) \cdot H_r(s) = \dfrac{s+1}{s^2-4}$.

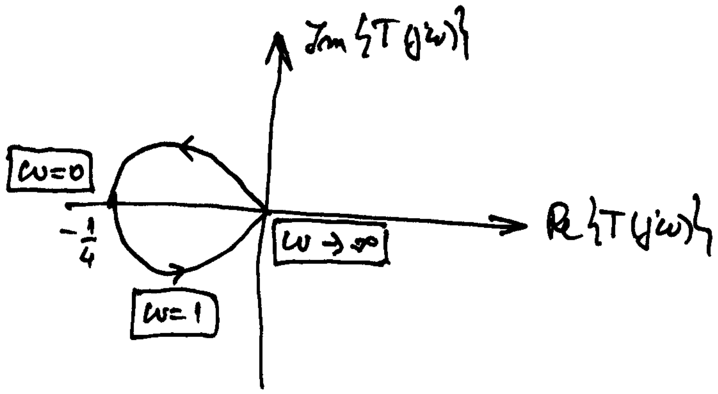
Polii lui $T(s)$ sunt: $s_{p_1} = 2$; $s_{p_2} = -2$. $\Rightarrow$ INSTABIL.

$T(j\omega) = H_d(j\omega) \cdot H_r(j\omega) = \dfrac{j\omega+1}{(j\omega+2)(j\omega-2)} = \dfrac{1+j\omega}{-\omega^2-4}$

$Re\{T(j\omega)\} = -\dfrac{1}{\omega^2+4}$ ; $Im\{T(j\omega)\} = -\dfrac{\omega}{\omega^2+4}$.

c)

| $\omega$ | 0 | 1 | $\infty$ |
|---|---|---|---|
| $Re\{\cdot\}$ | $-\frac{1}{4}$ | $-\frac{1}{5}$ | 0 |
| $Im\{\cdot\}$ | 0 | $-\frac{1}{5}$ | 0 |




Sistemul este stabil, dacă

$-\dfrac{1}{K} > -\dfrac{1}{4}$ ; $-\dfrac{1}{K} < 0$ $\quad \dfrac{1}{K} < \dfrac{1}{4}$ ; $K > 0$

Deci, în final avem:

$\boxed{K > 4}$

**Cod: S 04**

**REZOLVARE**

a) $H(z)\cdot G(z)=\frac{1}{z-\frac{1}{2}}$

$$H(e^{j\Omega})\cdot G(e^{j\Omega})=H(z)\cdot G(z)\Big|_{z=e^{j\Omega}}=\frac{1}{e^{j\Omega}-\frac{1}{2}}$$

$$e^{j\Omega}=\cos\Omega+j\sin\Omega$$

$$H(e^{j\Omega})\cdot G(e^{j\Omega})=\frac{1}{\cos\Omega+j\sin\Omega-\frac{1}{2}}=\frac{\cos\Omega-\frac{1}{2}-j\sin\Omega}{(\cos\Omega-\frac{1}{2}+j\sin\Omega)(\cos\Omega-\frac{1}{2}-j\sin\Omega)}$$

$$=\frac{\cos\Omega-\frac{1}{2}-j\sin\Omega}{(\cos\Omega-\frac{1}{2})^2+\sin^2\Omega}=\frac{\cos\Omega-\frac{1}{2}-j\sin\Omega}{\cos^2\Omega-\cos\Omega+\frac{1}{4}+\sin^2\Omega}=$$

$$=\frac{\cos\Omega-\frac{1}{2}-j\sin\Omega}{1-\cos\Omega+\frac{1}{4}}=\frac{\cos\Omega-\frac{1}{2}}{\frac{5}{4}-\cos\Omega}-j\frac{\sin\Omega}{\frac{5}{4}-\cos\Omega}$$

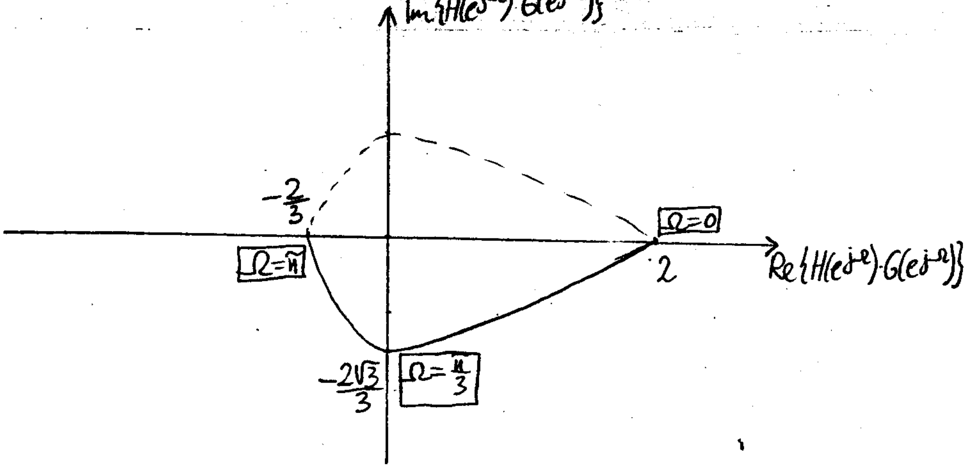
$$Re\{H(e^{j\Omega})\cdot G(e^{j\Omega})\}=\frac{\cos\Omega-\frac{1}{2}}{\frac{5}{4}-\cos\Omega}$$

$$Im\{H(e^{j\Omega})\cdot G(e^{j\Omega})\}=-\frac{\sin\Omega}{\frac{5}{4}-\cos\Omega}$$

b) $Re\{H(e^{j\Omega})\cdot G(e^{j\Omega})\}=0 \Rightarrow \cos\Omega-\frac{1}{2}=0 \Rightarrow \cos\Omega=\frac{1}{2}\Rightarrow\Omega=\frac{\pi}{3}$

$$Im\{H(e^{j\Omega})\cdot G(e^{j\Omega})\}=-\frac{\frac{\sqrt{3}}{2}}{\frac{5}{4}-\frac{1}{2}}=-\frac{\sqrt{3}}{2}\cdot\frac{4}{3}=-\frac{2\sqrt{3}}{3}$$

| $\Omega$ | $0$ | $\frac{\pi}{3}$ | $\pi$ |
|---|---|---|---|
| $Re\{H(e^{j\Omega})\cdot G(e^{j\Omega})\}$ | $2$ | $0$ | $-\frac{2}{3}$ |
| $Im\{H(e^{j\Omega})\cdot G(e^{j\Omega})\}$ | $0$ | $-\frac{2\sqrt{3}}{3}$ | $0$ |




Sistemul în buclă deschisă este stabil deoarece $|z_p|<1$.

Pentru ca sistemul în buclă închisă să fie stabil, trebuie ca hodograful construit să nu înconjoare punctul critic, de coordonate $(-\frac{1}{k}, 0)$.

$$-\frac{1}{k} < -\frac{2}{3} \quad \text{sau} \quad -\frac{1}{k} > 2$$

$$\frac{1}{k} > \frac{2}{3} \quad \text{sau} \quad \frac{1}{k} < -2$$

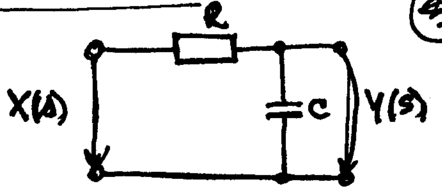
$$k < \frac{3}{2} \quad \text{sau} \quad k > -\frac{1}{2}$$

$$\boxed{-\frac{1}{2} < k < \frac{3}{2}}$$

## REZOLVARE

(30) (27)

a)



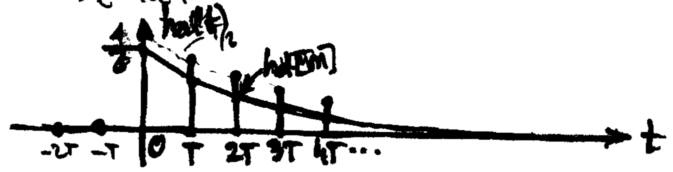

$$Y(s) = \frac{\frac{1}{sC}}{R + \frac{1}{sC}} \cdot X(s) \qquad H_a(s) = \frac{Y_a(s)}{X_a(s)} = \frac{1}{1+sRC}$$

$\tau = RC$ Rezultă $H_a(s) = \dfrac{1}{1+s\tau}$

$$H_a(\omega) = H_a(s)\Big|_{s=j\omega} = \frac{1}{1+j\omega\tau}, \quad H_a(s) = \frac{1}{\tau}\cdot\frac{1}{s+\frac{1}{\tau}} = \frac{\frac{1}{\tau}}{s+\frac{1}{\tau}}$$

$$h_a(t) \longleftrightarrow \frac{1}{\tau} e^{-t/\tau} \sigma(t)$$

b) $h_d[n] = T_e \cdot h_a(nT)\Big|_{t=nT}$, adică $h_d[n] = T_e \cdot \dfrac{1}{\tau} \cdot e^{-nT/\tau} \cdot \sigma[n]$




$$h_d[n] \overset{Z}{\longleftrightarrow} H_d(z)$$

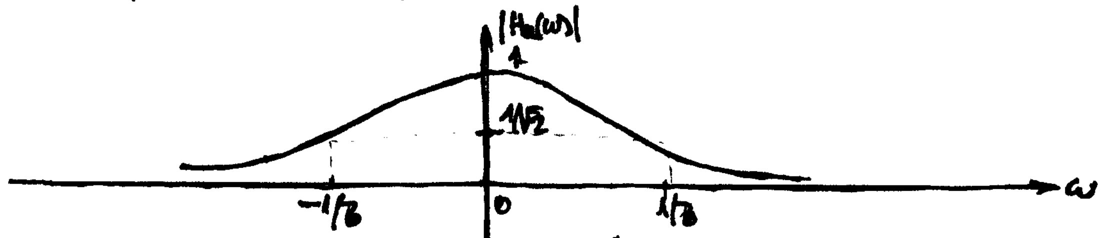
$$H_d(z) = \frac{T_e}{\tau} \cdot \frac{1}{1 - e^{-T/\tau} \cdot z^{-1}}$$

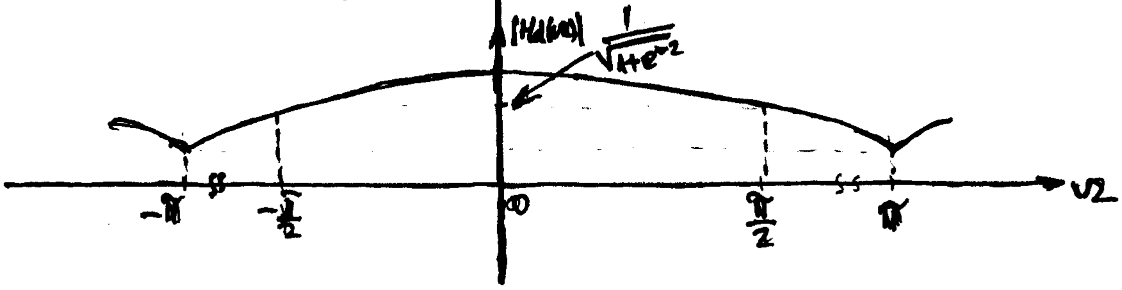
c) $$H_d(\Omega) = H_d(z)\Big|_{z=e^{j\Omega}} = \frac{T_e}{\tau} \cdot \frac{1}{1 - e^{-T/\tau} \cdot e^{-j\Omega}}$$

$$H_a(\omega) = \frac{1}{1+j\omega\tau} \qquad |H_a(\omega)| = \frac{1}{\sqrt{1+\omega^2\tau^2}} \qquad |H_d(\Omega)|\Big|_{T=\tau} = \frac{1}{\sqrt{1 - e^{-1} \cdot e^{-j\Omega}}}$$

$$e^{-j\Omega} = \cos\Omega - j\sin\Omega$$

$$|H_d(\Omega)| = \frac{1}{\sqrt{(1 - e^{-1}\cos\Omega)^2 + e^{-2}\sin\Omega}} = \frac{1}{\sqrt{1 + e^{-2} - 2e^{-1}\cos\Omega}}$$



(28.) a) $H_d(s) = \frac{10}{s+1}$ ; $H_r(s) = \frac{1}{s+10}$.

$$G(s) = H_d(s) \cdot H_r(s) = \frac{10}{s+1} \cdot \frac{1}{s+10} = \frac{10}{(s+1)\cdot(s+10)}.$$

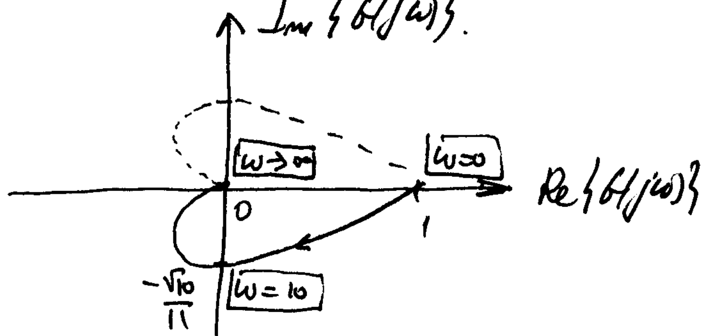
$s_{p_1} = -1$ ; $s_{p_2} = -10$.

Deoarece polii sistemului în buclă deschisă se găsesc în semiplanul stâng, rezultă că sistemul este stabil.

b.) $$G(j\omega) = \frac{10}{(1+j\omega)\cdot(10+j\omega)} = \dots = \frac{100 - 10\omega^2}{(1+\omega^2)\cdot(100+\omega^2)} - j\frac{110\omega}{(1+\omega^2)\cdot(100+\omega^2)}$$

$Re\{G(j\omega)\} = \frac{100 - 10\omega^2}{(1+\omega^2)\cdot(100+\omega^2)}$ ; $Im\{G(j\omega)\} = \frac{-110\omega}{(1+\omega^2)\cdot(100+\omega^2)}$.

| $\omega$ | 0 | $\sqrt{10}$ | $\infty$ |
|---|---|---|---|
| $Re\{\cdot\}$ | 1 | 0 | 0 |
| $Im\{\cdot\}$ | 0 | $-\frac{\sqrt{10}}{11}$ | 0 |

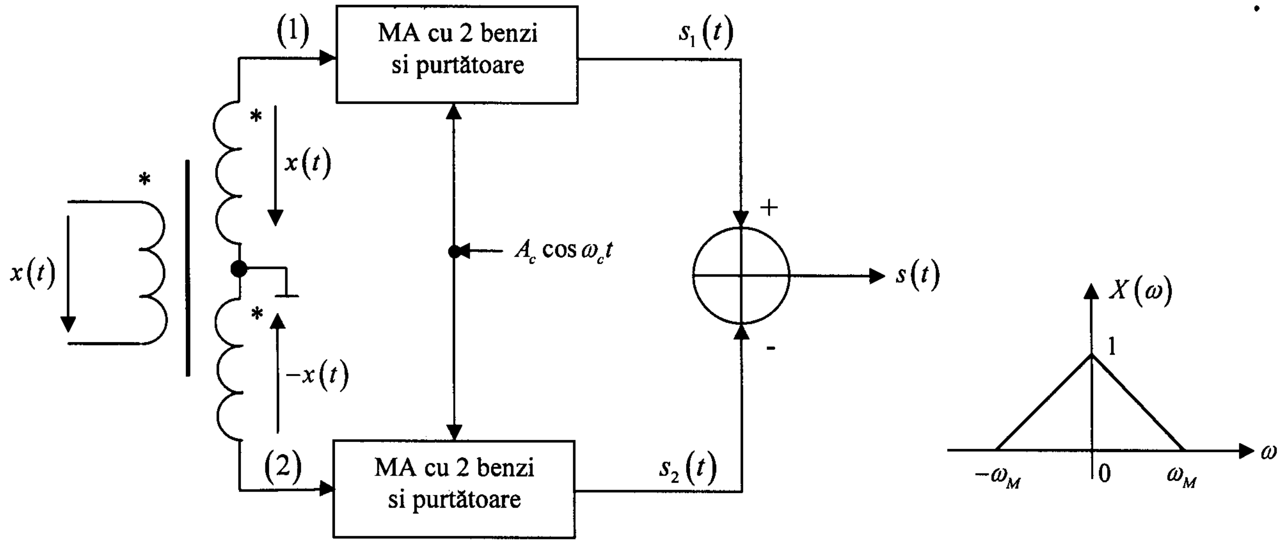



Sistemul în buclă deschisă este stabil.
Pentru ca sistemul în buclă închisă să fie deasemenea stabil, este necesar ca hodograful obținut să nu înconjoare punctul critic de coordonate $(-\frac{1}{K}, 0)$.

$-\frac{1}{K} < 0$ sau $-\frac{1}{K} > 1$

Rezultă : $\boxed{k > -1}$

**29**



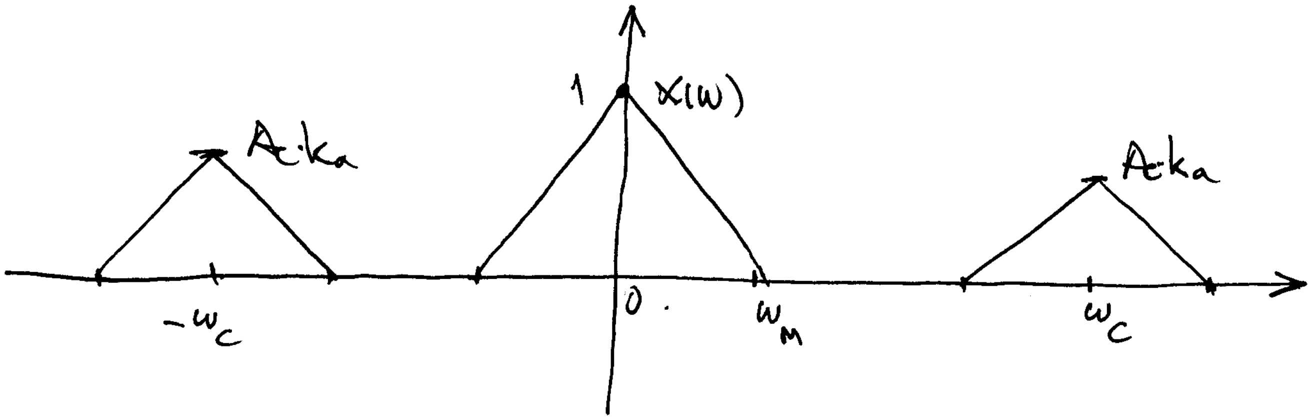

La ieşirea modulatorului de amplitudine (1) se obţine semnalul $s_1(t) = A_c[1+k_a x(t)]\cdot\cos\omega_c t$.

În timp la ieşirea modulatorului de amplitudine (2) se obţine semnalul $s_2(t) = A_c[1-k_a x(t)]\cos\omega_c t$.

Cele două semnale se scad pentru a obţine semnalul de ieşire al modulatorului echilibrat:

$$s(t) = s_1(t) - s_2(t) = A_c[1+k_a x(t)]\cos\omega_c t - A_c[1-k_a x(t)]\cos\omega_c t.$$
$$= 2A_c k_a x(t)\cos\omega_c t.$$

Spectrul $$S(\omega) = 2A_c k_a \frac{1}{2\pi} X(\omega) * \pi[\delta(\omega-\omega_c)+\delta(\omega+\omega_c)]$$
$$= A_c k_a X(\omega-\omega_c) + A_c k_a X(\omega+\omega_c).$$



REZOLVARE (30)

a) $\frac{dy(t)}{dt}+2y(t)=2x(t)$ /$\mathcal{L}$  $sY(s)+2Y(s)=2X(s)$  $H(s)=\frac{2}{s+2}$

$Re\{s\}>-2$  $H(\omega)=H(s)\Big|_{s=j\omega}$  $H(\omega)=\frac{2}{j\omega+2}$

Din tabel: $h_c(t)=2e^{-2t}\sigma(t)$

b) $h_d[n]=T\cdot h_c(nT)$  $h_d[n]=2T\cdot e^{-2nT}\sigma[n]=2T\left(e^{-2T}\right)^n\cdot\sigma[n]$

$H(z)=\frac{2T}{1-e^{-2T}\cdot z^{-1}}$

$e^{-j\Omega}=\cos\Omega-j\sin\Omega$

c) $H(\Omega)=H(z)\Big|_{z=e^{j\Omega}}=\frac{2T}{1-e^{-2T}\cdot e^{-j\Omega}}$

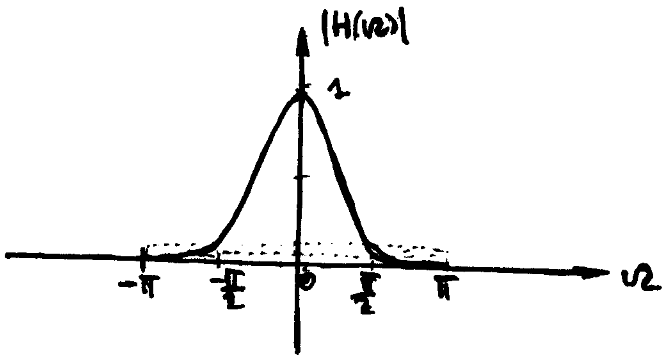
$e^{0,2}=1,22$
$e^{0,4}=1,5$
$T=0,1$

$H(\Omega)=\frac{2T}{1-e^{-2T}(\cos\Omega-j\sin\Omega)}$

$H(\Omega)=\frac{2T\cdot e^{2T}}{e^{2T}-\cos\Omega+j\sin\Omega}=\frac{0,2\cdot e^{0,2}}{e^{0,2}-\cos\Omega+j\sin\Omega}$

$|H(\Omega)|=\frac{0,2\cdot e^{0,2}}{\sqrt{(e^{0,2}-\cos\Omega)^2+\sin^2\Omega}}=\frac{0,2\cdot e^{0,2}}{\sqrt{e^{0,4}-2e^{0,2}\cos\Omega+\cos^2\Omega+\sin^2\Omega}}=\frac{0,2\cdot e^{0,2}}{\sqrt{2,5-2e^{0,2}\cos\Omega}}$

| $\Omega$ | 0 | $\frac{\pi}{2}$ | $\pi$ |
|---|---|---|---|
| $\lvert H(\Omega)\rvert$ | 1 | 0,15 | 0,1 |




d) $x[n]=\cos\frac{\pi}{2}n$

$y[n]=A|H(\frac{\pi}{2})|\cos(\frac{\pi}{2}n+\arg\{H(\frac{\pi}{2})\})$

$|H(\frac{\pi}{2})|=0,15$